

TREATISE
ON THE
PRACTICE, THEORY, AND HARMONIC SYSTEM
OF THE
VIOLONCELLO.

A SHORT ACCOUNT OF THE RECORDED ORIGIN OF MUSIC, AND OF ITS STATE
AMONG THE ANCIENTS.

THE derivation of the word *Music*, has been very variously accounted for. The Latin word *Musica* is derived from a Greek word of the same sound, signifying to *search*, or *seek out*. Others trace it to a Hebrew word meaning *art*, or *discipline*. *Musa*, and *Musica*, signified *learning* in general. The Athenians gave the name of *Music* to every art; and hence the Nine Muses, the daughters of Jupiter, who invented the sciences, are, by Poets and Mythologists, made to preside over these. Hermes Trismegistus (*thrice excellent*) says, that “Music is nothing but the knowledge of the order of all things.” This was the doctrine of the Pythagorean school, and of the Platonists, who taught that every thing in the universe is *music*. Pythagoras said, that he had a perception of the celestial harmony of the spheres; and Jamblicus reports of him, that he taught others to imitate this divine music, by instruments, and the voice. Aristides Quintilianus, and also Bacchius, in their relative treatises, term Music “a knowledge of singing.” Porphyrius, in his Commentaries on Ptolemy’s Harmonics, gives the subdivisions of the general science, as, *Harmonica*, *Rythmica*, *Metrica*, *Organica*, *Poetica*, and *Hypocritica* (relating to pantomimic gesture); applying *Rythmica* to dancing, and *Metrica* to enunciation.—The *Harmonica* is the only part of which the ancients have left any certain accounts; and even these are very obscure and imperfect. Lucretius writes, that the imitation of the *liquidæ avium voces* originally suggested the art of Music: and the invention of wind instruments arose from the whistling of the winds among hollow reeds. Music is as old as the world itself: the fable of the Muses proves its divine origin. Plutarch, in his Treatise on Music, ascribes it to Amphion. The Greek writers are divided on this point; some ascribing the origin to Amphion, some to Orpheus, while others ascribe it to Apollo.—The Sacred Writings are, however, the surest guides in this, as well as in almost all other ancient historical truths. Jubal, the sixth from Adam, is called, “the father of such as handle the Harp, or Organ.”—The *Lyre* of

seven strings was given, by Mercury, to Amphyon the Theban, who first taught the art of singing and playing simultaneously. Orpheus the Thracian introduced many arts into Greece, and particularly Music. Terpander the Lesbian, invented the *musical modes* applicable to Hymns, and quelled a mutiny in Sparta by the power of his Lyre. Thamyras the Thracian was the inventor of *Instrumental Music* without singing. He contended with the Muses, on condition that he should possess all their powers if he overcame, or lose what they pleased if vanquished. Being defeated, they put out his eyes, destroyed his voice, and struck him with madness. The ancients so little understood the compass of Music, that they fined Timotheus for adding a string to the Lyre. The ancient instruments were very imperfect: the *Tympanum* was a species of drum: the *Tibia* was made of the shank-bone of animals, as the crane: the *Fistula* was made of reeds: the *Syringa* consisted of seven reeds joined together, each reed affording a note: the invention is ascribed to Pan:

“ Pan primus calamos cerâ conjungere plures
Instituit.”

Vitruvius gives an obscure account of a species of Organ, called *Organum Hydraulicum*. The *Tubæ*, the *Cornua*, and the *Letui*, were, respectively, different kinds of Trumpets. The *Lyra*, invented by Mercury, was the principal stringed instrument: it was also called *Cithara*. It originally consisted of a *tetrachord*, or three tones, or notes, and a semi-tone. Three strings were added to the instrument, and these additions took place at various periods. The seventh string was, unquestionably, the suggestion of Terpander. Pythagoras raised the Lyre to the compass of a complete octave. He invented the *monochord*, and, by means of the proportional vibrations of its aliquot divisions, he accurately adjusted the intervals of the tone—the sesquialtera, the sesquitercia, diapente, and diapason—known better, in modern Music, under numerical appellations indicating their relative positions within the octave. The octave, similar to our division in these times, was divided into two parts or *tetrachords*. The ancients placed the semi-tone as the second note of each tetrachord, whereas the moderns place it last in each. When we treat of the *Harmonics of Strings*, it will appear, that all the *diatonic notes* of the fourth or last practicable octave of a string are *distinct harmonics*; that the interval between the first and second, and between the second and sharp third, constitutes a full tone; and that the semi-tone lies between the third and fourth. This being the natural interval of musical notes, and that by which even an uncultivated voice is known to rise in the scale, it follows, that the modern octave is *more naturally* divided than the ancient; and that this beautiful series of *harmonic notes* of the *fourth octave* cannot be deemed the effect of blind chance, and must be resolved into some correspondence with harmony beyond the limits of human conception. Subsequent to the improvements introduced by Pythagoras, the strings of the Lyre were, at various times, increased in number, as far as to include above four octaves.

To the *Cithara* may be traced the *Chitarra* of the Italians; the *Quttara* of the Arabs; the *Guitarra* of the Spaniards; and the English *Guitar*. In all these we observe the principle of the *Cithara* applied to a finger-board. The *Harp*, also, is evidently derived from the Greek *Lyre*; and the modern *Harpsichord* (modified into the *Piano Forté*) is nothing more than a *Harp disposed horizontally*, and yielding sounds by the mechanical contrivance so generally prevalent.

The *Cithara*, modified into the form of a *Harp*, was the instrument by means of which Saul “was refreshed, and was well, and the evil spirit departed from him.” This suggests the powerful use to which music might be applied in the cure of melancholy and insanity. In St. John’s Vision, the *elders* are represented with *harps* in their hands: and surely this, independent of ancient scriptural usages, may be deemed sufficient sanction for using Music generally in human worship, to elevate the mind, and to sublimate its conceptions. If even infants are soothed and lulled to rest by the imperfect efforts of an uncultivated voice, the sublimest effects may be expected from the powers of concordant harmony, regulated by the application of science, on principles unknown to the ancients.—Among the Greeks and Romans, a person devoid of an ear, or talent for music, was deemed *stupid*; and it was a reproach to be unable to play on their instruments. A *wise man, adorned with the graces*, was reckoned so, principally, in proportion as he was *Citharæ callens*. Aristotle termed Music the *medicine of heaviness*: and Horace calls his *Lyre, laborum dulce lenimen*. The Wise Man said, that “wine and music cheer the heart.” Music was the discipline by which Pythagoras, in a great measure, formed his scholars. He asserted, that the soul consisted of harmony; and that it brought into the world the memory of the music with which it was entertained in Heaven. Socrates learned to play in advanced life; and Plato, in his “Commonwealth,” recommends the use of music in society; being of opinion that it had a similar influence over the mind, to that which air has over the body. Anciently, laws, and exhortations to virtue, were sung musically.—Horace’s elegant panegyric on Music has been imitated, without being equalled*.

“Silvestres homines, sacër interpresque deorum,
Cædibus et victu fædo, deterruit Orpheus :
Dictus ob hoc, lenire tigres, rabidosque leones :
Dictus et Amphion, Thebanæ conditor arcis,
Saxa movere sono testudinis, et prece blandâ
Ducere quò vellet.” &c. &c. DE ARTE POETICA.

* The Romans cultivated the art of singing with an enthusiastic assiduity. The execrable tyrant Nero was in habits of lying on his back, with a plate of lead on his stomach, in order to improve his voice. He frequently took *emetics* and *cathartics*, and abstained from certain fruits and meats, with a view of preserving his voice. He had an officer, termed *Phonascus*, whose duty it was to regulate his tones.

The power of Music in war, has been felt by all nations. To this truth Virgil bears elegant testimony :

“ Quo non prestantior alter,
Ære ciere viros, Martemque accendere cantu.”

Pythagoras first measured the differences of acute and grave notes, by the ratios of numbers. He states, that if strings, in all respects alike, are stretched by weights, as 6. 8. 9. 12. they yield the *octave*, between the 6. and 12.; the *fourth*, between the 6. and 8. and between the 9. and 12.; a *fifth*, between the 6. and 9. and between the 8. and 12.; and that between the 8. and 9. there was exactly the difference between a *second* and *third*. Galileo found that not these weights, but their squares, must be used to produce those intervals. Strings of the above lengths, but otherwise similar, will furnish the intervals stated.—The ancients defined a *concord*, “the agreement of two sounds pleasant to the ear, either successively or jointly heard.” Of the simple concords they admitted only the fourth, called *diatessaron*; the fifth, called *diapente*; and the octave, called *diapason*. They had three genera: *Enharmonic*, consisting of subdivisions of a note; *Chromatic*, consisting of semi-tones; and *Diatonic*, consisting of the octave, divided into the two *tetrachords* already mentioned. The first and second genera were deemed of very difficult execution.—Their *modes* were principally three, *Doric*, *Lydian*, and *Phrygian*; and the essential difference between them, consisted in the *gravity* or *acuteness* of the whole octave. Of their mutations, the principal was the *Melopœia*, which was something similar to our *Minore*, changing the whole air, from gay and sprightly, to soft and languishing. *Melopœia* was also the art of making melodies, or songs. They expressed notes, by letters of the alphabet shaped imperfectly, or placed perpendicular, or horizontal, according to the number of notes used.

The ancients understood the art of *modulation*, which they denominated *Mistio*. By this they meant modulations carried into chords and keys different from the fundamental, and a subsequent return to the original key. Ascending passages were termed *ductus rectus*; and descending, *ductus revertens*. Euclid and Aristides treat of this part of Music with great clearness. The whole of the third book of Aristoxenes is designed as rules for the formation of melody. Composition in parts, as *treble*, *tenor*, and *bass*, or different melodies conjoined into one harmony, are no where mentioned by any of the writers alluded to.—Instrumental concerts, as now constituted by the rules of *counterpoint*, did not prevail in former ages; but concerts of voices certainly did. Aristotle, in his treatise concerning the world, writes, “Musica acutis et gravibus sonis, longisque et brevibus una permixtis in diversis vocibus, unum ex illis concentum reddit.” This passage is decisive as far as respects concerts of voices. The ancients undoubtedly joined instruments and voices together in one symphony; but each had not a distinct melody forming a general concordance by any other rule than the mere unisons of octaves. The modern art, where the parts differ by the different relations of the successive

notes arranged by rules for the various instruments, the ancients do not seem to have possessed. Seneca writes, “Non vides quam multorum vocibus chorus constet? Unus tamen ex omnibus sonus redditur: aliqua illic acuta est, aliqua gravis, aliqua media. Accedunt viris fœminæ, interponuntur tibix, singulorum latent voces” (no particular voice is distinctly perceivable) “omnium apparent.” Cassiodorus writes, “Symphonia est temperamentum sonitus gravis ad acutum, vel acuti ad gravem, modulamen efficiens, sive in voce, sive in percussione, sive in flatu”—*Symphony, in vocal or instrumental music, is an adaptation of grave and acute notes constituting a modulation.* All this proves nothing farther than that they sung and played in *choirs*, in probably octaves, and an unvaried junction of common concords, devoid of the multiplied and scientific intervals, demanding so much of that genius and cultivated taste apparent in modern composition.

The instrument termed *Pandora*, had four strings struck by a *plectrum*, or quill attached to the fore-finger. These strings furnished a fundamental, the octave, a fourth, and fifth. This was little better than *unison*; and was incapable of the compass and variety furnished by the comprehensive *modern style, in parts*. The public laws of Greece discountenanced every thing like innovation in their primitive and simple music: and this greatly accounts for the little knowledge possessed by the ancients in the art of *harmony and composition*, which may be, probably, too complex and elaborate, substituting too frequently a wild and unmeaning rapidity, or trick of finger, for a pathetic harmony addressed to the feelings, and calculated either to soothe or agitate the passions. It is highly probable, that modern composition, which so frequently steps out of nature into mere frivolous movement, captivating only to the eye, will, ere long, undergo an advantageous change; and thus lose the reproach of scientific nonsense applicable to a large portion of modern music. We are falling fast, in these respects, into a vitiated taste of admiring only what we deem difficult, while we forget the legitimate ends of music, *viz.* its charming effect in exciting the finest emotions of the mind, on subjects involving the best interests of virtue, and human happiness.

ON THE STATE OF MUSIC IN THE MIDDLE AGES;

WITH

A SHORT SKETCH OF THE GENERAL PRINCIPLES WHICH REGULATE IT IN MODERN TIMES,
AND SOME NOTICES OF INSTRUMENTS.

IN all ages, Music seems, very properly, to have formed no unimportant part of religious worship. About three hundred and fifty years after Christ, St. Ambrose introduced the *chaunting of the Psalms* into the Western Church. These *chaunts* were composed from four, to eight parts. Instrumental music was soon afterwards added. In the year 757,

Constantine sent an Organ, as a present, to Pepin King of France. In the dark ages; literature and refined studies were confined chiefly to the Court of the Roman Pontiffs: hence, in music, Italy has been to Europe, what Greece was formerly to Rome. The invention of counterpoint is given to Guido Aretinus, a Benedictine monk of Aretium in Tuscany, about the year 1024. It was called *contra-punctum*; because notation in music was first marked by dots. Counterpoint was used in secular music in the thirteenth century. In this century, John de Muris, or rather his master, Magister Franco, invented the *Time-Table*. The spaces and lines determining the relative positions of notes, are generally ascribed to Guido above-mentioned, early in the eleventh century. These Notes, marked by letters, he termed *Clefs*, or *Keys*. He is also supposed to be the inventor of *Spinets* and *Harpsichords*. The first Treatise on Composition in Parts, was published at Valladolid in the year 1570, by Thomas à Sancta Maria. It related principally to the composition for five instruments, called *Fantasia*, having four crotchets in a bar. The *Sonata*, and *Concerto Grosso*, followed soon afterwards: and, indeed, nothing superior to the Concerto Grosso, in point of real harmony, has appeared in the last or present century. The *Jongleurs* and *Menestrels* were privileged performers in the dark ages: they sung and played the poetic and musical compositions of the Troubadours. From the misconduct of these descriptions, they afterwards fell into disgrace. The principal instruments were the Harp and Viol, which latter was furnished with frets. It had six strings; these were reduced to four; and this instrument evidently gave rise to the modern *Violoncello*, called originally *Violone*. The *bow* derived its origin from the Arabs, who had it from the Eastern nations, among whom its use had been known from time immemorial. The *bow* must have been used by the ancients, in producing notes from the monochord. The *Double Bass* of our time, resembles, on an increased scale, the *Rebec* of three strings, which preceded the *Violin*. In the legendary life of St. Christopher, as early as the year 1200, the *Fiddle* is mentioned. On account of its imperfect construction, its tones did not establish any favourable character for it, till a more perfect form rendered it a concert instrument, early in the sixteenth century*. The Violins were termed Treble, Contra-alto, Counter-tenor, and Bass-violin. Our present Second Violin takes the part of the contra-alto. The part of the counter-tenor is now given to the tenor: and the part of the original

* James the First, of Scotland, was the inventor of a species of music which continues much admired at the period we live in. Alessandro Tassoni, and other authors, bear testimony to the musical talents of James. Tassoni writes;—"and among the moderns we may reckon King James of Scotland, who not only composed many sacred pieces of vocal music, but also of himself invented a new kind of music, plaintive and melancholy, different from all others; in which he has been imitated by Carlo Gesueldo, Prince of Venosa, who in our age has improved music, with new and admirable inventions."—James introduced Organs and Choir Music into abbeys and cathedrals in Scotland. It is highly probable that the beautiful and loyal national air of "God save the King," was composed by James. It was published in a book of Anthems at Aberdeen, in the year 1632. It is decidedly, in style and pathos, very similar to the affecting melodies of Scotland.

tenor is given, differently arranged, to the modern tenor and Violoncello. *Sonatus* were originally composed for the church. They afterwards became, under a lighter construction, the favourite music at concerts, till Guiseppe Torelli, of Verona, invented the *Concerto Grasso*. His Concertos were published in 1709; and those of Corelli, in 1712. Charles II., in imitation of the numerous performers in the concerts at the French Court, conducted by Lulli, introduced a similar practice in England; where the number of performers has gradually increased to what we now witness.—Stanley and Avison in England, and Geminiani and Tartini in Italy, excelled in the composition of the *Concerto Grosso*. Handel followed a similar plan in his *Grand and Hautboy Concertos*, with this difference, that he added wind instruments, and increased the number of performers. Handel's Overtures, so justly admired, have their ground-work in Corelli's compositions, though the plan and construction are very similar to the works of the celebrated Lulli. A wonderful change took place in musical composition, about the middle of the last century. The majestic and beautiful style of the eminent composers enumerated, was succeeded by what is called, with great propriety, *Modern Music*. It is very shewy, and abounds with difficulties of execution, the conquering of which requires a life-time, and seems to be almost the only reward it confers on its indefatigable and assiduous scholars. Whether this gratification of unwearied labour compensates for the evident frequent absence of the pathos and affecting harmony of past times, we leave it to more adequate judges to determine. At the same time, it is far from being meant, that *Modern Music* does not number among its composers men equally celebrated by the science of their works, and the beauty and taste displayed in their various styles. It may not, probably, be too much to say, that the rapidity of the *Modern Music* has attained its acme; and that this invention of a man of the name of Stamitz, may undergo some modification, calculated to re-conduct it to the approved and confirmed standard from which it has gradually departed.

It is not the intention here, to give any detailed account of the *Elements and Principles of Music*. It will, however, be necessary to state the doctrine *generally*, and as far as it may be connected with the *Theory and Practice of the Violoncello*, the nature of which cannot be thoroughly understood, without some description of the Harmonics of Strings. What will be found here, will call the attention of the reader to works where these interesting subjects are handled more at length.

Melody is a succession of agreeable sounds. *Harmony* is an union of two or more sounds, bearing such a relation to each other, that all the sounds constitute a coalesced one, termed *harmony*. Various modern authors have endeavoured to account for the nature and principle of Harmony. Rameau and Tartini are among the clearest foreign expositors of this subject, though Rameau is deemed too fanciful in reducing his theory to practice. The former says, that if a string be put in vibration by a bow, the fundamental sound, and its twelfth and seventeenth major, will be heard at the same time. This is a fact, and the experiment is most convincingly made on the third

string of a Violoncello. (See *Plate, Fig. 1.*) The open string sounded is G. The twelfth major D, and the seventeenth major B, will be heard distinctly, if the vibration of the string is *gently* continued. The twelfth major D, is the octave to D, the fifth, to G; and the seventeenth major is the sharp or great third to G, the double octave above G the open string. It is therefore evident, that B and D, the seventeenth and twelfth, are harmonics, which may be reduced to B and D, the great third and fifth to the fundamental, or open note G. If the seventh minor to G, or F natural on the third string, is gently sounded, the experiment will succeed, if possible, better; and the octave F, and twelfth major C (at the extremity of the finger board), will be heard clearly in consonance, and separately, particularly the C.—By proportioning the lengths of strings in the proportion of the fundamental—twelfth and seventeenth—Rameau also ascertained a similar correspondence in a descending gradation. The whole system of Thorough-bass is, in a great measure, founded on Rameau's theory.—Harmonics are notes produced without pressing the string. For instance, divide the third string into *five equal parts*: let the string be put in vibration; and let these points of division be touched by the finger *without pressure*, the seventeenth major B, will be produced by touching at each point, and furnish a fine clear note. The first division from the nut of the instrument, pressed by the finger, is the B, or the sharp third to G, the open note. From this it must follow, that the general vibration of a whole string, must, at the same time, contain within its compass or length, the vibrations of other aliquot parts. For instance, when the whole string is sounding the open note G, the third part of the string from the bridge is sounding or vibrating the twelfth D; and the fifth part from the bridge is vibrating the seventeenth, or B.

Tartini found, that if two different sounds are drawn from two instruments of the same sort, their coalescence will generate a third sound. This experiment is best made by means of two wind instruments well tuned, and placed at an equal distance from the person making it. From this generating principle of two concordant acute sounds, the lowest diapason of organs has been produced, on the Continent, without having recourse to pipes of the length of above thirty feet*.

Having mentioned Thorough-bass above, it is proper to remark, that this continued bass, is a fundamental bass, whose chords are inverted to adapt it to singing, and to take

* Two wind instruments, such as hautboys perfectly tuned, are the most eligible for making this beautiful experiment; and the hearers should be situated at an equal distance between them. Stringed instruments are not equally well calculated for the purpose; because the vibrations of different strings, as well as those of the body of the instruments themselves, intermix too much with the new or generated notes. From the fifth, a sound unison with the lowest generating sound is produced. The fourth produces a new sound, an octave lower than the highest note used. The third major produces a note an octave lower than the lowest note made use of. By this wonderful means, sounds, or perfect notes, are produced, that are double and triple octaves below the lowest generating original note.

off the monotony of a plain bass. A continued, or thorough bass, constitutes in fact a treble, in reference to the bass, which is too simple to form a modulation.

Monsieur Perreault, by making experiments with very long strings, ascertained, that while the whole string is vibrating, its various aliquot parts vibrate at the same time. In the sound of the fine notes termed *harmonics*, a delicate and well-cultivated ear will discover a considerable difference between the same note sounded harmonically, and produced by the pressure of the finger. (See *Plate, Fig. 1.*) The twelfth major, or note D (the third string is exemplified, on account of the clearness of its notes) is a fine harmonic sound, at one *third part* of the string measured from the bridge. By pressing the finger on this point (or rather somewhat below it, on account of the obtuse shape of the top of the finger) the same note D is produced; but it is inferior in sweetness and softness, to the same note harmonically sounded. In producing the note harmonically, the third part between the finger and the bridge, and the other two thirds between the finger and the nut, vibrate each at the same time. The vibrations of strings have been luminously treated of by Dr. Brooke Taylor, and the two celebrated mathematicians, the Bernouellis; and the former particularly, has clearly established the properties and nature of the harmonic curve, which has been farther illustrated by Dr. Smith, in his "Philosophy of Musical Sounds."

The nature of harmonic notes produced by the aliquot divisions of the string of a musical instrument, will be briefly animadverted to in a subsequent part of this small work; but, for a full demonstration of this interesting subject, the reader must be referred to the above writers.

Rameau has founded a system of harmony on the ground of concordant harmonics; and though late writers have treated his positions as visionary and impracticable, others are ready to allow, that thorough-bass and counterpoint may be reduced in a great measure to the principles laid down by Rameau. A sound from a bell, or a note from a string, will be still the same tone, whether produced by a greater or less stroke of a bow, or clapper. The only difference will be, a varying degree of loudness. On this is founded, the continued increase or decrease, termed *diminuendo*, and *crescendo*, which have so fine an effect in animating, or softening passages in music. Stringed instruments only are capable of giving fine harmonics. Their effect is destroyed in applying a violent *crescendo* to them; nor can their fine tone be much improved by any attempt to swell them beyond their natural and inherent fulness of tone.

Sounds are either *equal* or *unequal**.—When *equal*, they are called *unisons*.—When

* The connection between any note and its octave, is sufficiently evident from their perfect coalescence, when well sounded or tuned. Another very striking circumstance indicates this surprising and pleasing analogy. If a person possessing a good voice, but not habituated to music, intends to change from some pitch taken too much either above or below his voice, he will *naturally* take the *octave*, either higher or lower than he originally attempted. To take his pitch at any intermediate note, will demand a considerable effort or degree of attention. In joining in singing any tune, the

unequal, the difference between them, or their *distance*, is called an *interval*. A *concord* is that which, heard in succession of sounds, or in consonance or compound sound, has an agreeable effect. A *discord* in all its relations, shocks the ear, and has, in the above respects, a contrary effect. Concord and harmony, are in general, synonymous terms. A *compound interval* includes within itself two or more *simple intervals*. For instance, the difference between G and A, third string, is a tone major; between A and B, a tone minor; between B and C, a semi-tone; between C and D, a tone major; between D and E, a tone minor; between E and F sharp, a tone major; and between F sharp and G, a semi-tone. These notes, in their order, are the great second, the great third, the fourth, the fifth, the great sixth, the great seventh, and octave; and any one of them contains as many intervals as it is distant from the key note, G. These notes, respectively, may be yielded by a string divided as follows, reckoning from the bridge. Lay off $\frac{8}{9}$ for the point of the great second; $\frac{4}{5}$ for that of the major third; $\frac{3}{4}$ for that of the fourth; $\frac{2}{3}$ for that of the fifth; $\frac{3}{5}$ for that of the great sixth; $\frac{8}{15}$ for that of the great seventh; and $\frac{1}{2}$ of the string for the position of the octave. Take $\frac{5}{6}$ of the string from the bridge, and the position of the minor third, B flat, will be had. Take $\frac{4}{7}$ of the string from the bridge, and the position of the minor seventh, F natural, is had. This minor seventh is perfectly conformable to the *harmonic* (F natural) sound, above described, at one seventh part of the string from the bridge, being a double octave above F, the minor seventh to the key G. This minor seventh is, however, played a little different from the harmonic pitch of its double octave. It is the medial point between concord and discord; and is the favourite chord of Modern Music. It is, however, but a chord of substitution and borrowed harmony. Take $\frac{5}{8}$ from the bridge, and that point will be the position of E flat, the lesser sixth to G. The fourth, C, is also sharper than it is generally played. This *harmonic* fourth C, as it appears in the *Figure*, is $\frac{3}{32}$ parts of the string from the bridge. It is, however, played flatter in the proportion of 3 to 2.9.—The greater sixth E, is also played rather flatter than its harmonic position in the fourth octave indicates. It lies at a point in the first octave $\frac{3}{5}$ parts from the bridge. The next octave E, is $\frac{3}{10}$ from the bridge; the next octave, or third octave, lies at a point $\frac{3}{20}$ from the bridge; and the highest octave is at a point $\frac{3}{40}$ parts from the bridge. This great sixth is played a little lower down, or flatter, than at its real harmonic point, in the proportion of nearly 3 to 3.07.—No other answer than the predominance of custom, can be given to the question, why the reduced seventh is played sharper, and the fourth and sixth greater, flatter than the real

octave above, or below, is frequently taken; and the person so taking it, thinks he is singing in unison. There is also a singular analogy between male and female voices, which farther shews the natural affinity of the octave to its fundamental. If a man is singing, and a woman present joins in the song, she will quite naturally sing an *octave higher* than the man. In common church music, this effect is readily observable.

harmonic divisions authorise. Probably an easier rule for temperament of instruments with keys might be found, by restoring these notes to their just positions in the scale*. The notes instanced above, are referred to one string; but they may be drawn from strings disposed according to the following rule. *The number of vibrations made in the same time, by two chords of the same matter, differing in length, thickness, and tension,*

* Various divisions of the diatonic scale have been calculated by eminent musicians, with a view to form as perfect a chromatic scale as can possibly be constituted. On account of the inequality of the ratios of the scale, there ever must remain some notes false in a small degree. If the fifths and fourths are tuned perfect, the thirds and sixths will be somewhat false; and it is found that the flattening of the fifths is the most tolerable procedure. A scale divided according to the following ratios, is, probably, the most perfect that can be formed.

G.—G sharp.—A.—A sharp.—B.—C.—C sharp.—D.—D sharp.—E.—F natural.—F sharp.—G.

In this division of the scale, the fifths and fourths, except three, are true. Of the thirds less, and sixths greater, there are as many false as true; and both, thus, in a very small degree. As to the thirds greater, and sixths less, there are five false, and seven true. All these differences are so small, that the ear will easily bear them, more especially in the imperfect concords of sixths and thirds. But if several of these inequalities occur near to each other in a composition, the effect on a nice ear will be, that the instrument will seem to be out of tune, as the melody will appear broken, or interrupted. It is well known that some pieces run better on keyed instruments, than the same pieces taken on a different key. The reason of which is, that the inequalities of the semi-tones must necessarily be less in some keys, than in others.

The above imperfections are remedied, in a great degree, on stringed instruments, by *varying* the touch, or pressure on the string, under the guidance of a well cultivated ear. For instance, on the Violoncello, if A, the second note on the third string, be stopped a true sixth to C the open note of the fourth string, the A will not be a true fourth to D the open note of the second string, or the ninth to the note C. In this case, the finger must be moved forward *a little beyond* A; and the very small space it has moved over is termed *comma*, or about the fourth of a note, which, though an interval scarcely distinguishable by a common ear, still gives rise to the error in the general scale, which requires a subdivision of it, termed *temperament*, on keyed instruments. The Violin has only four fixed sounds, or the notes sounded open on the four strings; as *g* on the fourth, *d* on the third, *a* on the second, and *e* on the first string. These are tuned perfect fifths to each other. Here the interval from *d* to *a* is a true fifth; but in the diatonic scale it wants a comma. In stopping *a*, the second note on the fourth string, a greater tone 8:9 is placed between them, in order that there may be a true octave from *a* on the fourth string, to *a* the open note on the second string; but in the scale, the intervals have a fixed character. It is however a query, how far the stopping of the Violin is as accurate, as this amounts to, particularly in the more rapid passages; for it is found, that even on stringed instruments, some pieces of music answer better in some keys, than in others. This would seem to imply, that the notes are almost in all cases taken nearly in the same manner.

In the delineation of the finger-board of the Violoncello, given in *Fig. 1.* the chromatic divisions might have been marked according to some one or other of the theories regulating the subdivisions of a musical string: but for simplicity's sake, and to render the positions of the semi-tones distinct, they are marked as usual, or nearly at the semi-distances between the diatonic full notes.—It must, however, be always recollected, that G sharp, and A flat, are not exactly one and the same note. There is a proportional difference evident from the little that could be here said on the subject; and a good ear, aided by practice, and some knowledge of the *rationale* of the subject, will soon learn to take the notes very nearly in tune.

are in the compound ratio of the diameters, and lengths inversely, and the square root of the tensions, inversely. (See Malcolm's Treatise on Music.)

Within the octave there are seven concords, and two discords. The concords are the result of frequent (nearly) *union* vibrations, and *coincidences* of vibrations; and the discords of a rarity of coincidence. *Simple* harmony is produced by the combination of the concords within the octave. The harmony is *compound*, when the concords of two or more octaves are conjoined. The seven concords of the octave afford *eighteen combinations*: the fifth and octave: the fourth and octave: the sixth and octave: the third greater, and octave: the third less, and octave: the sixth less, and octave: the third greater, and fifth: the third less, and fifth: the fourth, and sixth greater: the third greater, and sixth greater: the third less, and sixth less: the fourth, and sixth less: and these last six pairs of concords sounded, respectively, with the octave.—Of these combinations, the last six are the most harmonious; and those where the fourth is found, the least pleasing. It is on this account, that Descartes calls the fourth *infelicissima*; and says it loses its sweetness by being so near the fifth, which is most perfect next to the octave. If the fundamental and fifth are sounded in consonance, the fourth above the fifth (being the octave to the fundamental) will also resound. On this account, the fourth is not admitted next the bass, but all the other concords are, because they answer their primary purpose of varying the fifth, which the fourth, from its natural alliance to the fifth, could not effect, and would be thus misplaced, and produce an unpleasing harmony.—Sir Isaac Newton happily discovered that the breadths of the seven primary colours in the sun's image produced by prismatic refraction, are proportional to the differences of the lengths of the eight musical strings; and it is remarkable, that the fourth colour in the order has the least pleasing effect, when compared, in contiguity with the others, respectively, or with their combinations.

The *harmonical intervals*, constituting concords, would not afford a sufficient variety in musical composition, without the use of what are termed *concinuous intervals*; for, otherwise, a movement from one concord to another, would be little better than a repetition of the same thing. The *concinuous intervals* principally used, are the ratios 8:9, a greater tone; 9:10, a lesser tone; and 15:16, a semi-tone. These are called *degrees*, or steps that agreeably connect the greater intervals with every advantage of variety, and without having recourse to the harsh *inconcinuous intervals*. Positive discords are admitted, where a strong contrast, or a surprising effect, is required in the general harmony; and the passages where they are used, are immediately resolved, or re-conducted into the harmonical intervals. The two thirds and the two sixths, have too small a ratio to each other, as 25:26, to be admitted both together into divisions of the octave; and, therefore, the following is the allowed arrangement. (See Plate, Fig. 1.)

G fundamental; B flat $\frac{5}{6}$ or third less; C fourth $\frac{3}{4}$; D fifth $\frac{2}{3}$; E flat sixth less $\frac{5}{8}$; and octave $\frac{1}{4}$ of the string.

G fundamental; B third greater $\frac{4}{3}$; C fourth $\frac{3}{4}$; D fifth $\frac{2}{3}$; E great sixth $\frac{3}{2}$; and octave $\frac{1}{6}$ of the string.

These degrees, sounded in various consonances, are good harmony ; but the third less, and sixth greater, would together make very nearly, a discord. Every note is a discord to the one following it ; but by substituting an interval, concordance is produced. The second A, and seventh greater F sharp, though agreeable as successive notes, are discords in combination ; but are still efficient in many situations, from what has been stated : and the discord does not offend the ear, on account of its immediate contiguity to preceding and following concords.

The above arrangements of the octave form *modes* in music, or the internal constitutions of the octave ; and the *fundamental note* in which a *close* is made, is termed the *key note*. The fundamental G, the third minor B flat, or the third major B, and the fifth D, are called *essential notes*. The other notes are denominated *dependent*. The fifth D is called the *dominant*, as having the most perfect affinity to the key. The third is called the *mediant*, from its situation between the *final* and *dominant*. The seventh is common to all the modes. Flat keys admit of both species of the seventh. The greater seventh makes a smooth passage into the key ; and is applied in making transitions from one key into another, particularly contiguous to *cadences*, when the subject is prepared to undergo a change. From its expressive utility, it is sometimes called the *sensible note*. The twelve intervals of musical sounds are (See Fig. 1.) : second lesser A flat, or G sharp ; second greater A ; third lesser B flat ; third greater B natural ; fourth C (called also *subdominant*, being under the *dominant*) ; false fifth, or tritone, C sharp, or D flat ; fifth D ; sixth lesser E flat, or D sharp ; sixth greater E natural ; seventh lesser F natural ; seventh greater F sharp, and octave G. The octaves of these are, simply replications, or repetitions of the same sounds, all commencing from the fundamental G.

The octave and fifth are called *perfect concords*. The two thirds, and two sixths, are called *imperfect concords*. The false fifth, the two seconds, and two sevenths, are discords. The fourth, on account of its situation between the third and fifth, can seldom be used as a concord, excepting when joined to the sixth. It is generally classed with the discords, though in modern music, it is used frequently in deviation from its legitimate description. It is a singular fact, that the human voice rises from the fundamental, to the second greater ; the third less, or greater ; the fourth ; fifth ; sixth less, or greater ; and greater seventh, into the octave ; and that if it has taken the *third less* in ascending, it never fails to take the *sixth less* in passing on to the octave. On these accounts, these seven notes are termed the *natural notes*, or *scale*. The key is denominated sharp, or flat, according as the third and sixth may be greater or lesser. Melody is the produce of imagination ; but harmony is founded on approved rules, and depends on the exercise of a cultivated judgment. The intervals in the treble, ought to be as small as possible. The bass may proceed by larger intervals. Ascending by a false fifth is harsh in effect, and is therefore to be avoided. To proceed by a spurious second, or from a note that is sharp, to the one immediately above or below, that is flat (and the reverse), is extremely offensive to the ear. The key may have its octave, third, or fifth. The fourth and fifth fundamental may have their respective thirds or fifths. The

sixth fundamental may have its third, fifth, or sixth. The second fundamental, third and seventh fundamental, may have their relative thirds or sixths. When the bass *ascends*, the treble must generally *descend*; and contrarywise. When the parts move the same way, upwards or downwards, two octaves, or two fifths, must not follow immediately. Two lesser sixths must never follow each other. Thirds and sixths are admitted repeatedly, and in immediate succession. *Simple counterpoint* admits of nothing but concords. In *figurate counterpoint*, the discords are admitted. The discord prepared, is applied to the unaccented parts; but the accented must have a full harmony. This, by foreign composers, is called *supposition*; because the transient discord supposes a concord to follow, to take off the effect of the discord. The discords are in music, what strong shades are in painting,—they make the concords appear to better effect. Discords must be followed by concords, and are called the *resolution of the discords*. The discord is first a concord to the bass note immediately preceding that to which it is a discord; and this is termed, *preparing the discord*. The discord is resolved, by being immediately followed by a concord descending from it in a small interval.

Every piece of music has a particular key, on which, or in which, it commences and terminates; but musical variety requires that the harmony should be frequently changed into other keys, to be re-conducted into the primary key. If the composition be long, medial cadences are used, previously to the termination by a final cadence in the key. In these transitions, there must always exist an analogy or harmonic connection between the assumed, and the original key. This is termed *modulating* in composition. Any key has seven natural notes, and the remaining five notes of the scale are deemed extraneous, and in general not adapted to purposes of modulation. In general, any of the seven natural notes may be constituted a *modulating key*, provided the third to it is some one of the original natural notes. This rule holds generally, but there are exceptions; particularly in modulating into a flat key. In a sharp leading key, the cadence is made on the key itself; then may follow, according to the nature and length of the piece, cadences on the fifth, third, sixth, second, fourth; concluding with a cadence on the principal key. In a flat principal key, the medial cadences are made on the third, fifth, seventh, fourth, and sixth. The greater seventh is the third greater to the fifth fundamental of the key; and by means of it, the cadence into the key is effected; for after the seventh is heard, the octave is expected to follow in natural order. On this account, a transition into another key is generally made, by introducing its greater seventh, which forms a smooth passage into the key to which the modulation is to be made. By an attentive inspection of the works of the most eminent composers, as Corelli, Handel, Haydn, Mozart, &c. &c. the nature of modulations and cadences will be best perceived, as exemplification here would lead far beyond proposed limits.

A DELINEATION AND EXPLANATION OF THE WHOLE OF THE FINGER-BOARD OF THE VIOLON-CELLO ; AND OF THE HARMONIC SYSTEM OF THE INSTRUMENT, INCLUDING THE DIATONIC AND CHROMATIC SCALES, TO THE EXTREMITY OF THE FOURTH OCTAVE OF EACH STRING : AND THE NUMERICAL DIVISIONS CORRESPONDING TO THE POSITIONS OF THE TONES AND SEMI-TONES WITHIN THE COMPASS OF THE RELATIVE OCTAVES OF THE GENERAL SCALE.

FIGURE 1. exhibits a proportioned delineation of the tones and semi-tones of the harmonic and natural divisions of each string, up to the termination of its fourth octave. This (reckoning tones and half-tones) gives in all forty-two. The finger-board extends nearly to the note marked *nineteenth major*. This note is E, third octave on the first string. The next notes F, and F sharp, are not expressed ; because they cannot be played in that position, by reason that the finger-board does not extend to them. If it did, the whole of the four octaves would afford forty-four divisions expressive of the notes of the general scale, considered either as tones or semi-tones, on each string.

The dotted line, *Fig. 3d*, has marked on it the harmonical divisions of the string corresponding with the natural position of each tone and semi-tone. The description of the notes in this figure, is adapted to the note of the open string considered as a *fundamental tone*. Were the lowest semi-tone (for instance, C sharp on the fourth string) taken as a tonic note, the position of the second major, third major, and of all the described notes, would be removed higher, or to the semi-tone immediately above, or in reference to C sharp, the new tonic.

The semi-tone minor, or F natural on the third string, is, by natural position, situated at $\frac{9}{16}$ of the string from the bridge ; but a delicate ear always takes it *a little lower*, or on the line nearly half way between $\frac{9}{16}$ and $\frac{4}{7}$ of the string. There is even some inclination to take it at the harmonic point $\frac{4}{7}$, corresponding to the twenty-first minor seventh of the third octave. There is also an inclination to take the tritone a little lower than usual, or at the harmonic point $\frac{5}{7}$. The same remarks hold in the second octave. The position of the sixth minor is $\frac{5}{6}$ from the bridge : nevertheless, it is well known, that a cultivated ear, from force of habit, takes it somewhat lower, or on the line on which the semi-tones are marked. It is to be observed, in general, that in stopping the notes and half-notes, the finger is to be applied near to, and rather below, the various *diatonic* and *chromatic* divisions ; as otherwise, there would be a liability of stopping *too sharp*, on account of the shape of the top of the fingers. This is particularly to be attended to when playing the *harmonics*. For instance, the *twelfth major* E, on the first string, is a *natural harmonic* ; and is so called, because it can be sounded either *harmonically*, or by *pressure* on the point E. When, however, it is played by pressure, the finger must be pressed on the string a *little lower* than the point E. This remark is applicable to harmonics compared with the note at their position, got by pressure.

The situations of the *octaves* to the *fundamental note*, are marked by a heavy line drawn across the strings of *Fig. 1*. The situation of the *fourth* to the open note, is also marked by a heavy line. It is to be observed, that each octave is half of the extent of the octave

immediately below it. For instance, from the lowest *second major* D, on the fourth string, to its octave D, in the second octave, $\frac{4}{9}$ of the string are included; and from D in the second, to D its octave in the *third octave*, is included the half of this distance, or $\frac{2}{9}$. The numerical line, *Fig. 3d*, shews that this rule is applicable throughout the entire extent of the finger-board, and to the extremity of the fourth octave.

The vibrations of the aliquot parts of the string, yielding the seven concords of the octave, will stand thus:

Unison	1	:	1	Sixth greater	5	:	3
Octave	2	:	1	Third greater	5	:	4
Fifth	3	:	2	Third lesser	6	:	5
Fourth	4	:	3	Sixth lesser	8	:	5

To explain this, let a string be divided into two equal parts, as at A the middle of the first string, the whole string A M will make *one vibration*, while its half A A (*ceteris paribus*) will vibrate *twice*. A similar remark applies to the vibrations of the parts of strings of equal substance, kinds, and thickness, measured off in the above proportions, as shewn by *Fig. 3d*.

The following simple experiment will prove, that the aliquot parts of a string vibrate the same harmonic note, while the intermediate points are at rest, and produce no sound. Let the second string of a violoncello be divided, as per *Fig. 4th*, into six equal parts. If the fourth finger is gently applied at A in the third octave, the *harmonic nineteenth major* will sound. Keeping on the fourth finger, let the thumb be applied at A (an octave below), another sixth of the string, the *nineteenth major* will still sound. Let a second person apply a finger at the middle of the string, being another sixth of the string farther down, the *nineteenth major* will still sound. If, at the same time, the assistant applies a finger at A, the position of the fifth major, and another finger at F, the position of the third minor, still the *nineteenth major* will sound. The fingers, as described, being kept thus on each aliquot, or sixth part of the string, if the bow is applied successively, *between* the position of *each finger*, the same *nineteenth major* will sound on *each* sixth part of the string experimented on thus separately. It must be quite evident, that the positions of the fingers, or the points they rest on, including that of the thumb, must be at rest, while the intermediate portions of the string vibrate the *nineteenth major*: for if the bow be applied to the string, at *any* of the positions of the fingers (moved off to make room for the bow) there will be no vibration or harmonic, because the point the bow is applied on, is a *point at rest**. Supposing the fingers and thumb to remain as described, let all be

* In making the above experiment, a curious fact discovered itself; and from this it appears quite possible to play on stringed instruments, or on a monochord, without the use of a finger-board. All vibrations are made in a *straight line*. If the finger is pressed against the point of the string marking any note, so as to make the string form a small angle there, the vibration below that point is as completely interrupted as if such point were pressed on the finger-board. The reader will find, that a tune may be perfectly played without stopping a single note, by actual pressure. Harp players may, probably, avail themselves of this circumstance, not, we believe, hitherto noticed.

removed from the string excepting the thumb, and the finger placed at A the fifth major, or $\frac{1}{5}$ part of the string from the nut. In this state of things, let the bow be drawn, and instead of the *nineteenth major*, the *twelfth major* A in the second octave, will immediately sound; because the string is now harmonically touched, at each *third part of its length*. If in this state of things, the bow is drawn, successively, between the bridge and the first third part A from that end; between the nut and the first third part A from that end; or between A and A; in each of these cases, the same *twelfth major* A, will sound. If the thumb is removed from A, it will also sound. If the thumb is put down again at A, and the finger raised from A, it will equally sound the *twelfth major*. Suppose the thumb and finger down as described, and that the *twelfth major* is sounding, should the string be designedly, or accidentally touched, the note sounded will immediately stop, which affords the clearest proof, that, though each *third part* of the string is, *at the same time*, vibrating the *same note*, the separate vibrations are connected, or concatenated through the points A and A relatively at rest. If the same experiment be made relative to the *seventeenth major*, and its corresponding divisions, as marked, at each *fifth part* of the string, the result will be similar. It is well known, that in this instance, the fundamental or open note sounds: therefore, it must be concluded, that while the aliquot parts separately and conjointly sound a harmonic, the whole string, or fundamental of that harmonic, is also sounding, however faintly: for the points mentioned to be at rest, are so only in reference to the vibrating aliquot parts, as they have a movement occasioned by the simultaneous vibration of the whole string*. The double octave, and all the harmonics of the fourth octave, vibrate on the principle thus reduced to a simple and beautiful experiment. The fourth octave might be equally had at the end of the strings next the nut, were a bridge placed there in lieu of the nut. But on account of the closeness of the strings to the finger-board, the harmonic of the twenty-second major D, is the last that can be brought out distinctly at that extremity of the instrument.

When the Violoncello, Tenor, and Violin are properly tuned, the harmonic A, at the middle of the Violoncello first string, is unison with the open first string of the Tenor; and the twelfth major E on the first string, is unison with the open first string of the Violin. A very accurate mode, therefore, of tuning the Violoncello, is, to screw up its first string, till the harmonic E sounds unison with the open E, first string of the Violin. The principle of this is, that the ear judges more accurately of unisons, than of octaves†.

* Let a long string be covered with thin silver wire. Let it be put in vibration. If *gently* interrupted at the two third parts of its length, the string will divide into three similar vibrations, while the whole will continue to have a general vibration; and thus, *at the same time*, the *fundamental* and *twelfth major* will be sounded sensibly to the ear. If a twelfth be sounded near an open string, it will resound to it, and also to a fifteenth or seventeenth major.

† Independent of the division of the Violoncello strings into consecutive octaves, it appears from this, that still more distinct divisions may be supposed. The first octave being *under* the tenor

In making the experiment with the aliquot divisions of sixths (any other harmonic divisions, as fourth or eighth parts, might be equally used) of the string, if the exact middle points between the sixth parts are touched by assistants, while the sixth parts remain also gently touched, the action of the bow will bring out the fifth major *a*, of the fourth octave, of the second string, being an octave above A the nineteenth major. In this case, the points of the string stopped, are twelfth parts of the string, corresponding to the position of the harmonic note *a*, one twelfth part of the string from the bridge. All the other harmonic notes of the fourth octave depend on a similar principle. As the aliquot parts of a string sound in unison, with isochronous vibrations of each; and sound, each, the *same note*; it may be supposed that the harmonic note ought to sound louder than the same note arising from pressure. Though the vibrations are isochronous, there is still a *diminuendo* in the effect downwards from the aliquot division where the action of the bow is applied. The note sounded harmonically is certainly louder, but the increased sound is so tempered and softened by the blended and progressively softened sounds of the aliquot parts, that the difference in loudness is not immediately perceivable. If a note is sung, and is sounded in unison by a few other weaker voices, the absolute degree of sound is increased, while, at the same time, the hearers are scarcely sensible of this, at least in any degree proportioned to the additional voices.—The limits of this short work will not admit of explaining the harmonic system more at large; and it is hoped that the Violoncello player will fully comprehend its nature from the experiments stated, and which he can repeat in a variety of ways leading to the same conclusion. He must always recollect, that the twelfth and seventeenth major, when reduced, are the great fifth and third of the fundamental note; and that in applying these to practice, in accompanying keyed instruments, the imperfection in them arising from *temperament*, must be strictly attended to; as the same piece played alone, must be played somewhat differently when keyed instruments are introduced. This truth every violin player is equally sensible of.—The most scientific treatises on Music have been written to produce a system of mean tones, to render the harmony equal in all keys, so that the sharp of the note below, may be the flat of the note above. This is the most difficult proposition in music; and the most advantageous division of the inequalities of intervals, has exercised the genius of the most eminent writers on the subject. It is well known, that four consecutive fifths compose a greater interval than two octaves and a third major, by the difference called *comma*; and no better mode of equalizing the scale has been hitherto found, than that of diminishing each fifth by the fourth part

pitch, is properly denominated the *lower compass*. The first note of the second octave, being unison with the tenor open strings, is called, in the following system of fingering, the *tenor pitch* of the instrument. This continues through four notes, or up to the twelfth major unison with the open strings of the Violin, and termed *Violin-pitch* in the account of fingering. This pitch contains an octave. The fourth octave may be termed *Altissimo*.

of this comma*. It is on this account, that some compositions answer better in some keys than in others; and that some keys, such as A, appear brilliant, while others, like F, appear more simple. These are unavoidable defects, or imperfections in keyed instruments; and the Violoncello, to be heard to advantage, must be adapted to such circumstances.

We shall now proceed to explain the system of Notes of the finger-board of the Violoncello, specifying distinctly the various positions of the same note, and how often it can be found both by *pressure*, and *harmonically*, in different situations on the four strings. This is so much the more necessary, as many tolerable bass players are not in habits of considering this; and confine their common accompanying practice to the lower compass of the instrument, when the same passage might be played with a finer expression and effect in the middle compass, and at the extremity of the lower compass. This species of knowledge of the finger-board, is more especially requisite for the Solo player, and in cases of compound accompaniments, where rapid transitions of the hand must be made along the whole extent of the finger-board. The harmonics beyond the fourth octave are so faint, that they are not marked off; besides, their coincidence of vibration with the notes below, is so distant, that their concordant use would have a harsh effect.

See Fig. 1. The note *c* at the extremity of the fourth octave on the fourth string, is C of the third octave of the first string. In the same manner, *g*, the last note of the second string, is unison with G a fifth above the last mentioned C. Also *d*, the last note of the third string, is *d* in the fourth octave of the fourth string, or a fifth above the last mentioned G. This shews, that the notes *c*, *g*, and *d*, the last of their respective strings, are situated also, respectively, at the distance of an octave and a sixth; an octave and a second; and a fifth from *a*, the extreme note of the first string. This consideration furnishes a ready reference to the relative situations of lower notes, according as they may be situated on the different strings.

The first four notes on the fourth string, as C, D, E, and F, can be found *only* where they appear. The next four notes, G, A, B, and C, on the fourth string, can be also found as the first four notes of the same appellations on the third string; because the interval between C and G, is a fifth major, corresponding to a similar tuned distance between the strings. The next four notes, D, E, F, and G, on the fourth string, can be played in two other situations;—*viz.* as the fifth, sixth, seventh, and octave of the ~~second~~ ^{7th} string; or as the first, second, third, and fourth notes, D, E, F, and G of the second string. This description carries us as far as the twelfth major, natural harmonic G,

* Let a violin be hung on a wall with its neck downwards, with one of its strings tuned unison with the middle C of the Piano. Let another string be tuned unison with this string, by suspending weights to it, till this effect is produced by the application of a bow. Add now to the weight, in the proportion of the square of eighty to the square of eighty-one; the additional part will be nearly a fortieth part. The difference of tones will be now very harsh; and will be precisely that termed *comma*.

on the fourth string. It appears, that this G can be played harmonically, in three situations;—*viz.* in its position as twelfth major; at the point G, one third part of the fourth string from the nut; and at the middle of the third string, at G. It appears, also, that C, the fourth of the third string, can be played as a fine harmonic, at C, the middle of the fourth string. Excepting these two, C and G, none of the notes as yet specified, can be played as harmonics unison to their note.

The next seven notes following the twelfth major G, on the fourth string, *viz.* the notes A, B, C, D, E, F, and G, can be found and played, each, in four distinct situations: that is, in succession on the fourth string; in succession from G, the middle of the third string; in succession from the same note G, the fourth on the second string; and in succession, beginning with the open note A of the first string.—Let it now be ascertained how many of these last mentioned notes can be played *harmonically*, and in what relative situations. A and B, the first two of these seven notes, have no unison harmonic. C, the third minor on the first string, is the double octave of the open note of the fourth string, and is an unison harmonic at that point; and also, as marked, at the point F, a fourth part of the fourth string from the nut. This C, therefore, can be played harmonically in two places. The next note in succession, D, the fourth on the first string, can be played harmonically in three places;—*viz.* at D, the middle of the second string; at D, as twelfth major to G, the open third string; and at D, the octave under this twelfth major, on the third string. The next note, E, or fifth major on the first string, can be played harmonically in four places on the fourth string;—*viz.* at each fifth part of that string, the highest being E, the seventeenth major on that string, and the other three positions on that string, being marked, “Harmonic of the Seventeenth Major.” The next note F has no unison harmonic. The next note, G, can be played as an unison harmonic to itself, in four different places;—*viz.* at the point G, double octave, third string; at the point C, fourth major on the same string; at the point G, nineteenth major, fourth string; and at the sixth part of the fourth string, from the nut, marked, “Harmonic of the Nineteenth Major.” The next note after the nineteenth major G on the fourth string, is A, at $\frac{3}{20}$ of the string from the bridge; but it is not marked in *Fig. 1*, because the finger-board does not extend under it. It can be played *by pressure* in three places;—*viz.* at A, the middle of the first string; at A, the twelfth major of the second string; and at A in the the third octave of the third string. This note A, is a fine unison harmonic in three situations;—*viz.* at A the middle of the first string; and at A, and A, the fifth and twelfth major of the second string.—We are now arrived at B flat harmonic twenty-first minor on the fourth string. This harmonic is found at each seventh part of the string, as marked, “Harmonic of the Twenty-first Minor.” The B natural, twenty-first major on the fourth string, cannot be played there, because the finger-board does not extend to it. This B natural, however, can be played *by pressure* in three places;—*viz.* as the ninth note of the first string; as the thirteenth note of the second string; and as the seventeenth major on the third string. It can also be played as a fine unison harmonic

at each fifth part of the third string, marked, "Harmonic of the Seventeenth Major." From this it appears, that the B flat can be got in six places harmonically, and in three by pressure: and that the same B, the natural, can be got in three places by pressure, and in four situations as a harmonic unison. The description of the notes is now carried on to *c*, the first note of the fourth octave on the fourth string. This note is also *C*, the third minor-compound of the second octave, first string. The description of the notes will be transferred to this *C* on the first string. This *C* can be sounded *by pressure* in three places;—*viz.* as described, on the first string; at the extremity nearly of the second octave, second string; and at the point *C*, the fourth of the third octave, third string. It can be played *harmonically* as *c*, the first note of the fourth octave, fourth string; and at the eighth part of the fourth string from the nut, marked in *Fig. 1*. "Harmonic of the Twenty-second Major."—The next note, *D*, the fourth of the second octave, first string, is procured by *pressure*, there, and as double octave of the second string. It is had *harmonically*, as double octave of the second string; as nineteenth major of the third string; and as second of the fourth octave of the fourth string. It is had also harmonically at *G*, the fourth part of the second string from the nut, and at the sixth part of the third string from the nut, marked "Harmonic of the Nineteenth Major."—The next note, *E*, the twelfth major of the first string, is had by *pressure* there, and at *E*, second string, third octave. It can be sounded in three places *harmonically*;—*viz.* at *E* as twelfth major, and at *E* an octave below it on the first string; and also as *e*, third major of the fourth octave, fourth string.—The next note, *F* natural, is had by *stopping* on the first string, and also in the third octave of the second string. The same *F* natural is procured *harmonically* at each seventh part of the third string, marked "Harmonic of the Twenty-first Minor." This *F*, as a sharp, is obtained four times *harmonically*—at each fifth part of the second string, marked "Harmonic of the Seventeenth Major;" and also as a harmonic at *f* (rather sharp), the fourth of the fourth octave, fourth string. This note is therefore obtained in seven places harmonically, and in two places by stopping, as a natural; and in four places harmonically, as a sharp: that is, in all, in thirteen situations.—The next note in the scale, *G*, is obtained by *stopping* it on the first string, and in the third octave of the second string. The same note is played *harmonically* at the eighth part of the third string from the nut; as first of the fourth octave, third string; and as fifth of the fourth octave, fourth string.—The following note, *A*, double octave, first string, can be played by *stopping*, only there. It can be played *harmonically* in six situations;—*viz.* at the double octave position *A*, and at the fourth part of the first string from the nut, *D*; at the the position of the nineteenth major *A*, second string, and at the position *F* on the second string, marked "Harmonic of the Nineteenth Major;" and also as *a*, second of the fourth octave, third string; and *a*, the sixth of the fourth octave of the fourth string.—*B*, the next note in succession, can be got by *stopping*, only on the first string. It is obtained *harmonically*, as third of the fourth octave, third string; and as seventh major of the fourth octave, fourth string.

—The following note, C, is only got by *pressure* on the first string. It is obtained *harmonically* in eight places, as C natural;—*viz.* at each seventh part of the second string, marked “Harmonic of the Twenty-first Minor;” as *c* (rather sharp), fourth of the fourth octave, third string; and *c*, the last note of the fourth octave, fourth string. The same note as C sharp, is obtained *harmonically* in four places on the first string;—*viz.* at each fifth part of that string, marked “Harmonic of the Seventeenth Major.” From this it appears, that this note C, considered as natural and sharp, is procured in one place by stopping, and in twelve places harmonically: in all, in thirteen places.—The next note, D, in the scale, is got in one place only, by *stopping* its position on the first string. It is had *harmonically*, as first of the fourth octave, second string; and as fifth of the fourth octave, third string. On a small second string, it may be sounded at an eighth part of the string from the nut.—The following note, E, is nineteenth major of the first string. It is played *harmonically* in three places;—*viz.* as nineteenth major E; at C, the third minor of the first string, marked “Harmonic of the Nineteenth Major;” at *e*, the second of the fourth octave, second string; and at *e*, the sixth of the fourth octave, third string.—The next note, F, in the third octave, first string, cannot be played there, because the finger-board does not extend under its position. It is played *harmonically*, as *f*, the third of the fourth octave, second string; and as *f*, the seventh of the fourth octave, third string.—The next note, G, marked “Twenty-first Minor Harmonic,” on the first string, cannot be played by stopping; but is had *harmonically* in eight places;—*viz.* at each seventh part of the first string, marked “Harmonic of the Twenty-first Minor;” as *g*, the fourth of the fourth octave, second string; and as *g*, the last note marked on the third string.—The next note, *a*, begins the fourth octave of the first string. It is a harmonic, and may be sounded *harmonically*, as *a*, the fifth of the fourth octave of the second string.—The next notes, *b*, *c*, and *d*, are *harmonics*; and are also harmonics as sixth, seventh major and minor, and concluding note of the fourth octave, second string—*e*, *f*, *g*, *g* sharp, and *a*, are also all *harmonics*, terminating the fourth octave of the first string.—Such is a concise account of all the notes and harmonics of the Violoncello.

Before this explanatory subject is quitted, it will prove satisfactory to project a figure, shewing, distinctly, the nature of the experiment before mentioned, and without a reference to the finger-board, excepting naming the notes produced.

See *Fig. 4th*. Let A, B, represent a Violoncello third string divided into six equal parts, as marked by letters. If the finger be gently applied at *m*, the first division from the bridge B, and if the bow be applied between the point *m* and the bridge, a very fine harmonic, D, the nineteenth major, will sound. If the thumb be applied at *n*, the next division (the finger remaining on *m*, as before), the same nineteenth major will sound. If the bow is applied between the points *m* and *n*, thus touched, the same note is still obtained. Let the finger and thumb remain lightly touching the string as before; and let an assistant gently touch the other three points of divisions of the string, *o*, *n*, and *m*; and the same harmonic note will sound, by bowing on *any* of the aliquot parts. From this easy experiment, it clearly follows, that *each sixth aliquot part of the string, is, at the same time, vibrating one and*

the same note. The points marked by letters, are *at rest* : for if a finger be taken off, and the bow is applied where the finger was, there will be no harmonic sound heard, but, on the contrary, a jarring harsh sound, arising from forcing the string into some irregular action. If, when the bow is applied on any of the aliquot parts, any point on any part of the others is touched, the harmonic sound will immediately cease. This affords a decisive proof that a concatenated vibration runs from one end of the string to the other, *through the points at rest*.—Let the thumb, and all the fingers engaged, be now raised from the string, excepting the fingers touching the points n and n , which mark thirds of the string; and let the bow be applied on any part of any of the third parts; and the sound of the twelfth major D , will be the predominant one; but evidently accompanied with the sound of its octave, the former nineteenth major, provided that the bow be not applied exactly on the point m , as that would prevent almost entirely the sound of the octave. If the five points marked by letters, have each a light thread tied round them, and if the finger be gently rested *close* to m , the nineteenth major, where one of the threads is tied, or close to any of the other points, the harmonic will sound, and the threads *will not* appear to vibrate. If, on the contrary, a small thread is tied on any part of any of the aliquot divisions, it will appear to vibrate strongly, when the nineteenth major is sounding. This experiment again proves, that the points marked by letters, are *at relative rest*, though the progress of the aliquot general vibration runs round them, or through them. If a finger is applied at o , only, the middle of the string, either lightly or by pressure, the octave to G , the fundamental note, will sound, accompanied by the nineteenth major, which is at the same time the twelfth major to G at the middle of the string. By listening attentively, while the bow is used with a slow, uniform, and steady movement, the note b , the great third of the fourth octave, and the seventeenth major to G , at the middle of the string, will be heard, as if at a distance. If the bow be applied at the point of the twelfth major, it will still be distinctly heard in consonance with the fundamental note.

If, instead of sixth parts of the string, fifth, or fourth parts of it are experimented on, the effect will be similar. The two fourth parts next to each other being lightly touched, the double octave to the fundamental G will sound; and when the bow is applied *between* the fingers touching these points, the same double octave will still sound. If each fifth part be bowed on separately, the seventeenth major to the fundamental will sound; or if any two or three (or four of these points of division) be touched simultaneously, the same seventeenth major will sound, wherever the bow may be applied. If threads be tied as before, the effect will prove as mentioned above. This *varied* repetition *establishes* the fact.

Let a , d , m , b , *Fig. 5th*, represent the third string of a Violoncello. The dotted harmonic curves, a , c , b , represent a vibration of the whole string a , d , m , b . It is well known, that while the string is vibrating the fundamental note G , the twelfth major D , and the seventeenth major B , are at the same time heard. In this case, while the whole string is vibrating, the third part, d b , must be vibrating the twelfth major, and the fifth part m b must be vibrating the seventeenth major. Not

only this, but also the remaining two third parts, and the remaining four fifth parts, must be, also, from our experiment, in vibration, *along* with the third and fifth part of the string nearest to the bridge. What has been explained under *Fig. 4th*, tends to prove, that this must be the case: for in sounding the fundamental note, if the bow is used on any part of the string, the twelfth major and seventeenth major are heard as before. Frequently, also, the octave, and nineteenth major, are heard along with these last mentioned notes. The fact is unquestionable; and whatever may be urged against it, the system of *general accompaniment*, advanced by the Bernouillis, and particularly by one of them, derives no small strength from the present fact, and from our experiment proving the separated and concatenated vibrations of the aliquot parts of strings. It appears, therefore, that much remains yet to discover, in order to establish, on clear principles, the theory of *universal accompaniment*. That lower notes are accompanied by higher ones, is an unquestionable fact. That these higher notes are produced by vibrations of known lengths of the *same string*, is also undeniable. That to produce those higher notes *along* with the fundamental one, there must be vibrations of one description, *within* others of a different description, giving another, or other accompanying notes, *is evident to sense*, by listening to the various notes yielded by the vibration of a long and thin monochord covered with silver wire. On the Piano Forte, the twelfth and seventeenth major are heard to sound when the fundamental note is dying away; and this will be the case if the wires of the twelfth and seventeenth major to the ground note, are prevented from vibrating by the power of sympathy, by applying the finger to them to stop all vibration. In this case, however, the twelfth and seventeenth major will not sound *as loud as before*; because the wires of the twelfth and seventeenth major, though tempered notes, still *sounded by sympathy*, and *increased* the sound of the same notes furnished by the wires of the fundamental note, in consonance with that note. It is altogether a most interesting and curious subject, which, like all complex human knowledge, though partly elucidated, appears to an inquiring mind, almost "past finding out." The controversy between Daniel Bernouillis, Euler, and D'Alembert, on this subject of compound and superinduced vibrations, on a chord considered as a moveable axis, remained undetermined at the death of the parties. The wonderful fact of *vibration within vibration*, or *superinduced vibration on a fundamental note*, is *evident* from a variety of simple experiments with chords kept in constant and strong vibration; but that the *rationale* of this astonishing phenomenon will be fully established, after the efforts of the first mathematicians have succeeded in part only, is more than problematical; and, at least, it must remain a subject of investigation and research, involving no inconsiderable difficulties.