INTRODUCTION

TO

THE SECOND PART.

THERE are various applications and definitions given by the Ancients of the word *Mufic*; but as they do not fall in with the plan propofed, I fhall confider Mufic in a lefs extenfive fenfe, and apply it only to Harmony and Melody.

Harmony is the fcience of founds, of which melody is a part.

Air

Air and motion, under certain regular meafures and proportions, are the efficient caufes.

To difcover what this motion is, and how to apply it to mufical founds, fo as to be able to know their agreement, difference, or mutual relation, is the object of Science.

It will be proper, as a preliminary, to explain the nature and properties of mufical ftrings, with refpect to their different lengths and vibrations, whofe agreement or difference is the object of the Ratio.

On thefe a foundation may be laid, from which a theory, the moft rational and agreeable to harmony, may be formed; one that will not only determine the proportion, agreement, or difference of founds, with refpect to acute and grave, but will teach us their ufe and application, fo as to anfwer all the demands that harmony or melody can poffibly make. But to arrive at this point of perfection, fome original leading principle, fome univerfal character muft be difcovered, whereby not only concords

cords and difcords, and their various combinations may be known, but their progreffions and fucceffions; for unlefs this very ufeful part of mufical knowledge can be determined by, and reconciled to, the laws of fcience as well as of practice, we fhould ftill want a faithful guide, an unerring principle.

It is not fufficient to know the feveral proportions of harmonical combination, we muft endeavour to difcover fome principle that will not only teach us how to form the mafs, but how to give it life and motion.

All founds are formed or generated by motion; that is, by percuffion or collifion; and as the air is the medium that conveys the found to the ear, if the motion ceafe, the found will alfo ceafe; and to whatever diftance the found is carried along this medium, fo far the motion paffeth, as it were in a fphere; according to the force with which the fonorous body, the centre, is acted upon.

The parts of a ſonorous body being thus put in motion, the trembling or vibration is either equal and uniform, or elſe unequal and irregular; and according to the conſtitution of the ſonorous body is ſwifter or ſlower: from whence ariſes the difference of ſounds.

Sounds are concinnous, or inconcinnous; that is, ſuch as are proper, or improper, for muſic.

Again, they have the difference of acute and grave; which, like great and ſmall, are but relative ideas; each of them has a certain magnitude, but one only is great, and the other ſmall: thus if two ſounds of different dimenſions be compared, one is acute and the other grave.

This relation of acuteneſs and gravity is one of the principal and moſt intereſting parts of a muſical theory, and is properly the object of mathematical demonſtration.

The

The degrees of ſound may be infinite ; yet with reſpect to thoſe which are proper for muſic, they are limited.

All ſounds differing in acuteneſs and gravity conſtitute an Interval.

This interval may form a concord or a diſcord, according to the relation of inequality, or particular difference of tune. But in order to fix the degrees of tune, and meaſure their relations by certain determinate qualities, we muſt have recourſe to the *Vibrations* of elaſtic bodies and their different *Lengths*.

If two ſtrings of the ſame length, thickneſs, and tenſion, be put in motion, they will vibrate in equal times ; that is, they will move in the ſame meaſure, joining and uniting in every courſe and recourſe, keeping ſtill the ſame equality of motion and identity of tune, having no interval or ſpace between. Therefore the habitude of theſe two ſtrings is called uniſon, and has the ſame relation to ſounds that unity has to numbers, and is in the ratio of 1 to 1, or of 2 to 2, &c.

If two ſtrings are in all reſpects the ſame, except their lengths, which are in the proportion of 2 to 1, that is, when the grave ſtring 2 is as long again as the acute 1, the graveſt ſound, or longeſt ſtring, will make but one vibration in the ſame time that the acute, or ſhorteſt ſtring, makes two; and when compared together will make the interval of an octave, whoſe ratio is as 2 to 1.

If ſix ſtrings are taken, whoſe *Lengths* are in the following proportion, 1, 2, 3, 4, 5, 6, the next of any two next numbers will be the acute term, and the other the grave. But it will be the reverſe of this if the ſame numbers are applied to the *Vibrations*; for then the leſs of any two next numbers will be the grave, and the other will be the acute.

Hence we may obſerve, that the different lengths of chords are inverſe or reciprocal, as their different vibra-

vibrations; and we may equally apply to one what is found true or agreeable to the other. For as their lengths are increafed, fo their vibrations are in the fame proportion decreafed, and vice verfâ.

Here then we difcover *two diftinct characters* in mufical ftrings: the firft gives the ratios, which exprefs the intervals from the acute term to the grave, by comparing a fhorter ftring with a longer; the other gives the ratios, by comparing the vibrations of a grave with an acute found; and each of thefe operations begins at unity, and moves in a contrary direction.

As the ratios are the fame in each, it may feem very immaterial whether we defcend or afcend from a given found or pitch; but when it fhall appear that the formation *of the minor and major fcales* depends on thefe two oppofite properties in mufical ftrings, the importance of the application will fufficiently juftify the diftinction.

For the information of thofe who are unacquainted with the nature of thefe operations, I have

have at Plate X. fig. K, placed ſix lines, which repreſent ſo many ſtrings, whoſe different lengths from unity ſhew the different ratios of their intervals. The figures placed on the left of theſe lines 1, 2, 3, 4, 5, 6, indicate their different lengths; and thoſe on the right ſhew the muſical interval, as an octave, 12th, 15th, &c. with a letter annexed to each, the more readily to diſcover their relation to unity, whoſe pitch is fixed at B.

Thus from B to B is an octave in the ratio of 1. 2.

From B to E is a 12th, or compound 5th, in the ratio of 1. 3.

From B to B a 15th, or compound octave, in the ratio of 1. 4.

From B to G a 17th major, or a major 3d, twice compounded, in the ratio of 1. 5.

From B to E a 19th, or a 5th, twice compounded, in the ratio of 1. 6.

If we compare their mutual relations we have

2. 3.

2. 3. a fifth,	3. 4. a fourth,
2. 4. an octave,	3. 5. a sixth major,
2. 5. a tenth, or 3d major,	3. 6. an octave.
2. 6. a compound 5th,	4. 5. a major third,
	4. 6. a fifth,
	5. 6. a minor third,

And if the fourth string had been doubled, we should have had the minor sixth in the ratio of 5. 8, which would have compleated the whole number of consonant intervals; but as there are only three letters in the whole combination, I have limited the series to six numbers, in order to preserve the fundamental bass E in the grave, with its harmonics, a minor 3d and a 5th, in the ratio of 6. 5. 4.

By these harmonics, which are formed from the different *Lengths* of strings in an arithmetic series, we have the true original principle of the minor mode, which has, by descending from unity, a *minor* 3d, next the fundamental or gravest sound at E 6.

I shall now examine the *Vibrations* by the same six numbers, at Plate X. fig. L.

These,

These, like the former, proceed from unity, but in a contrary direction; for as the number of vibrations in a given time is expressed by the same series as the lengths of strings 1, 2, 3, 4, 5, 6, the slower vibrations are the grave, and the quicker are the acute.

Thus the string that vibrates once is an octave graver than the string which vibrates twice in the same time 1. 2; and for the same reason two vibrations will be a fifth graver than three in the same time 2. 3.

If we proceed with the vibrations, we have all the simple and compound intervals expressed by the same ratios from grave to acute, as they were before by the different lengths of strings from acute to grave; also the *major* 3d next to the fundamental, F, A, C, or 4, 5, 6; whereas the *Lengths* of strings placed the *minor* 3d next the gravest or fundamental sound, 6, 5, 4.

From these opposite properties in the same musical strings we discover the two species of thirds, and

and in them the eſſential principles for the formation of the minor and major ſcales, which in another place will be more fully explained.

I ſhall now make ſome obſervations on the production of conſonant intervals as they are derived from a ſingle vibration; by which the true character of a fundamental baſs and its harmonic powers will be found the great and leading principle of a theory of harmonics.

In examining the ſeveral intervals which proceed from unity, we find that notwithſtanding their great variety their effects are all directed to one point, in order to mark and diſtinguiſh one original, or primary, ſound, ſuperior to the others.

To illuſtrate this I ſhall take the whole ſeries, as ſet down in the harmonic tables, Plate XI. &c. which are formed in conſequence of the vibrations of ſtrings, and are the foundation of the ſecond diagram, as the different lengths are of the firſt.

Here we may obſerve how a given ſound at unity in generating its harmonics ſtrengthens itſelf, and increaſes its powers by its octaves; thus 1, 2, octave (after which its fifth 2, 3, exiſts in its way to its next octave 1, 2, 3, 4, which at the ſame time generates a fourth 3, 4, after the original ſound is reinforced by two octaves) proceeds to the imperfect concord the third major 4, 5, or (which is the ſame) its ſeventeenth major 1, 5, in its way to its nineteenth 1, 6, or octave to its fifth 3, 6, and finally finiſhes the whole creation of harmonics with its third octave, 1, 2, 3, 4, 5, 6, 8.

There are two other imperfect concords, viz. the minor third 5, 6, and the minor ſixth 5, 8; but as they are formed by the mutual relations, and not by the primary, they cannot be fundamental baſſes in a major ſcale.

In the operations above deſcribed it muſt be owned, that nature ſeems to proceed with the greateſt caution in not permitting the fifth and third to appear until the baſs advances to its different octaves; thus acquiring freſh power by their addition, it preſerves preferves the firft impreffion of a generator or fundamental bafs, which otherwife might be weakened and overpowered by hearing the fifth and third too foon.

It appears alfo that the fifth, the next in degree of perfection as a concord to the octave, is doubled in the feries, as 2, 3, and 4, 6; the fourth alfo (by the mutual relations, not fundamentally) is doubled as another perfect concord 3, 4, and 6, 8. But thefe fourths are only fo many octaves to unity, as 1, 2, 4, 8, and therefore are a great addition to its power.

It appears alfo, that as neither of the thirds or fixes is doubled, one may conclude this is a fufficient reafon for their being diftinguifhed as imperfect concords; and being only once heard in the whole combination, they are prevented from becoming predominant, or from diminifhing the natural powers of the fundamental bafs.

From thefe obfervations the character and perfection of a fundamental bafs muft be admitted as the original principle of harmonic combination;

 fince

ſince it contains within its octaves all the conſonant intervals, which, when united and ſung together, yield one of the moſt pleaſing and perfect combinations that can be produced by harmony. But this character does not abſolutely depend on its being accompanied with the whole creation of its harmonics, ſince they are ſuppoſed to exiſt in as full and ample a manner in F, A, C, or 4, 5, 6, as if the whole ſeries in the harmonic tables (which are comprehended in three letters to each root) had been applied: and into whatever poſition theſe harmonics may be tranſpoſed, as from F, A, C, to A, C, F, or C, F, A, or to any other form, yet F will be the original generator, though leſs powerful in proportion as the harmonics are further removed from their original place of gravity.

From what has been obſerved on this ſubject, it appears to a demonſtration how every fundamental baſs generates its own harmonics, either by the different *lengths*, or the *vibrations* of ſtrings.

It now remains to ſhew the relation and connection that, one fundamental baſs has with another, in in order to difcover the true principles by which the motions of the different harmonics are regulated fo as to conftitute melody.

The harmonics of a given found, or fundamental bafs, can have no motion, or change, but among themfelves, unlefs the fundamental bafs moves to another bafs, whofe relation to it is fuch as produces the moft agreeable alteration poffible among the harmonics.

What this relation is with refpect to thefe baffes comes now to be explained.

In the arithmetic feries, which has been applied to the different lengths of ftrings, or their vibrations, we find only *thefe three primes*, 2, 3, 5; but as the ratio 2, 3, is a more perfect relation than 4, 5, or 5, 6, it muft therefore have a preference in directing the motion of the fundamental bafs.

As the harmonics have been generated from unity, fo the motion of the fundamental baffes muft be directed from the fame point.

Thus

Thus if we take a given ſound as a principal generator expreſſed by unity, and compare it with 3, according to the different lengths of ſtrings, we have 1, 3, or a compound fifth deſcending; and by a like compariſon with their vibrations, we have alſo a compound fifth aſcending from unity. See Plate XII. where C is made the principal generator, and has an immediate communication with its two products, F its fifth below, and G its fifth above.

Theſe are therefore the firſt and primary motions of the principal generator, and their relations are the moſt perfect, as they form a geometric ſeries in the ratio of 3; for as the vibrations give 1, 3, aſcending, and the lengths of ſtrings give 1, 3, deſcending, the ratios will be equally expreſſed by continuing the ſeries, as 1, 3, 9; in which the mean, or ſecond, term is the principal, as unity was before: now if each of theſe three terms be multiplied by the arithmetic ſeries, or only a part of it, as 4, 5, 6, [Pl. XIII.] or 6, 5, 4, [Pl. XIV.] according to the ſpecies of third wanted next the fundamental, we ſhall have the harmonics of each,

from

from which the fyftems of tetrachord and diapafon are formed.

Hence it appears that a theory of harmonics depends on the *union* of a *geometric* with an *arithmetic* feries. The firft directs the moft perfect motion of the fundamental bafs, as from 3, 1, or 3, 9, and by the other we difcover the harmonics of each.

Thefe principles being applied to the lengths and vibrations of ftrings yield all the intervals proper to form the minor or major fcales; and if the geometric progreffion be continued in the ratio of 3, as in the harmonic tables, we fhall have an increafing feries of flat diefes by the different lengths of ftrings, [Pl. XIV.] and of fharps by their vibrations, [Pl. XIII.] together with that vaft variety of intervals proper for every mode and gender, and for the mutations.

But before I proceed, it will be proper to take notice of fome few operations of numbers that they may be more readily applied to mufical intervals.

As every muſical ſound ſtands in ſome relation to a given one, or unity, it follows that each muſt be expreſſed by its particular number in the harmonic tables, in order to ſhew its relation to unity, or to any other number, or ſound.

There are two ſorts, or ſpecies, of motion in muſical ſounds mentioned by the Ancients.

The firſt the diaſtematic, or moving by ſkip, which ſeems natural to the gravity of the fundamental baſs.

The other the ſyſtematic, which confined the ſounds to the natural degrees of voice, peculiar to melody.

To preſerve theſe ſpecies of motion diſtinct, two ſorts of progreſſion are required, which, as they are eſſential to this theory, are formed into harmonic tables.

The firſt is the geometric, which is a continued multiplication from unity in the ratio of 3; as as 1, 3, 9, 27, 81, &c. Thefe are placed in the margin on the left of each table, and ftand as fo many Roots, from which the harmonics are formed by the application of the fecond progreffion, that is, the arithmetic; which is placed in every column next the root in this feries 1, 2, 3, 4, 5, 6, whofe common difference is 1. See Plates XIII and XIV. To this progreffion the 8th is added, in order to extend the whole to the third diapafon, which comprehends, as has been obferved before, the whole variety of confonant intervals.

The mufical letters ftand in the next column, as harmonics to the root; and their relation to each other, or to the root, is determined by the arithmetic feries, which alfo expreffes the ratio of each interval.

Each root of the geometric feries is the multiplier of the arithmetic, and is the common difference of that feries: thus the root 3 is the common difference in this feries, 3, 6, 9, 12, 15, 18, the harmonics to that root, which are placed immediately after the letters. Alfo root 9 will multiply the arithmetic ſeries, and diſcover its harmonics, as follows: 9, 18, 27, 36, 45, 54. Hence we not only have their mutual relations, but thoſe which ariſe from comparing them with the harmonics of any other root, however remote.

The harmonics of each root, as thus deſcribed, are in their loweſt denomination, and the numbers parallel in the ſucceeding columns are their reſpective octaves, made ſo by multiplying the firſt, and every ſucceeding, product of the root by 2.

Thus by multiplying either of the primes 2, 3, 5, by 2, the product will be octave; and if continued, as in theſe tables, will be a ſeries of geometric proportionals in the ratio of 2: but as a ſeries of octaves, applied as baſſes or roots, can make no alteration in the harmonics, which will always be the ſame, the other two primes 3 and 5 will abundantly ſupply all that is neceſſary.

We have already ſeen the great variety which ariſes from a progreſſion in the ratio of 3; it remains now to proceed to the other, which is alſo geometric,

geometric, in the ratio of 5; thus, 5, 25, 125. This ſeries is alſo formed into harmonic tables; the firſt at Pl. XV, repreſents the different lengths of ſtrings, and correſponds with the firſt diagram: the ſecond agrees with the vibrations Pl. XVI, and correſponds with the ſecond diagram.

In theſe tables every root is multiplied by the arithmetic ſeries, 6, 5, 4, or 4, 5, 6, by which we have a minor third 6, 5, next the fundamental baſs in the firſt, and a major third 4, 5, in the ſecond diagram; and by adding 24, a multiple of root 3, taken from the triple progreſſion, Pl. XVI, we have two ſpecies of thirds in the harmonics of root 5, in each diagram; and by proceeding in the ſame manner with the quintuple progreſſion, we have the ſame two ſpecies of thirds to every ſucceeding root; and the ſemitone minor 24, 25, is not only the interval formed by theſe two ſpecies, but is alſo diſcovered in the quintuple progreſſion, between E 120 of root 5, and E♯ 125 of root 25, in the ſecond diagram.

As it is of the greateſt conſequence in a theory of harmonics to know the original of all intervals which proceed from unity, the better to diſcover their ſeveral relations, I ſhall mention one more: this is the enharmonic dieſis, which is formed by comparing 128, a multiple of root 1, Pl. XIII, with 125, root 25, in the quintuple progreſſion, Pl. XVII.

In theſe harmonic tables the quintuple progreſſion proceeds from 5, to 25, the next page parallel with it, and after to root 125, in the next or following page parallel with the former; and in the ſame manner the roots proceed from left to right, as 15, 75, 375, quintuple, or in the ratio of 5. But if they proceed from one root to the next in the ſame column, the progreſſion is in the triple, as 5, 15, 45, in the ratio of 3.

It muſt be obſerved, that the arithmetic ſeries, which multiplies the root, expreſſes at the ſame time the ratio of the products: thus, if we multiply 15 by 6, 5, 4, we have 90, 75, 60, which are in the ratio of 6, 5, 4.

Having

Having explained the principles on which theſe harmonic tables are conſtructed, it remains to ſhew the uſe of them in a theory of harmonics, by the following rules.

Rules relating to the RATIO.

The harmonic tables contain a ſeries, or progſ-ſion, of roots from unity in the ratios of 3 and 5, and the harmonics to each are formed by the arithmetic ſeries before deſcribed.

Rule I. Every root and its harmonics are multiplied by 2, by which we have a ſeries of ſimple and compounded octaves continued in the ratio of 2.

II. Hence it follows, that every compounded number in the ratio of 2 may be reduced to its loweſt denomination by a continued diviſion by 2; and the compounds of the roots of 3 or 5 may be reduced in like manner by a diviſion by 3 or 5.

III. The difference between one number, or term, and another, is the ratio, or difference, between

tween one found and another expreffed by thofe numbers.

IV. If one term be compared with another in large numbers, they may be reduced to a fimple ratio, by being placed as fractions and divided by a common divifor; thus $\frac{32}{30} = \frac{16}{15}$ the common divifor will be one of the primes 2, 3, 5. In fome cafes the lefs term is fubftracted from the greater, and the product will be the divifor; thus $\frac{75}{50} = \frac{2}{3}$ a fifth, or $\frac{48}{45} = \frac{16}{15}$ femitone major.

V. When two or more ratios are added together, they muft be placed as fractions, and the numerator of the firft, and the denominator of the fecond, muft be divided by a common divifor, till brought to their loweft denomination. Thus if we add a tone major and minor together, the fum will be a major third: $1\ \frac{\overset{1}{9}}{8} + \frac{10}{\underset{1}{9}} = \frac{5}{4}$; alfo $1\ \frac{5}{4} + \frac{6}{5} + \frac{8}{6} = \frac{2}{1}$. Thus the major and minor thirds being added to a fourth, the extreme founds are octave to each other. Alfo a comma 81. 80, added to a tone

tone minor 10. 9, is a tone major, $1\ \frac{\overset{9}{81}}{\underset{8}{80}} + \frac{\overset{1}{10}}{\underset{1}{9}} = \frac{9}{8}$.

VI. It happens ſometimes that we have no common diviſor, in which caſe the two numerators, and the two denominators, muſt be multiplied into each other, and if the product has no common diviſor, we are ſure it is in its loweſt denomination.

Thus a major third, and a tone major, added together, $1\ \frac{5}{4} + \frac{9}{8} = \frac{45}{32}$, form a greater 4th, or tritonus.

VII. If a minor third be added to a deficient third, we have a common diviſor only to one numerator and one denominator, $1\ \frac{\overset{2}{6}}{5} + \frac{32}{\underset{9}{27}}$. Here 6 and 27 are divided by 3, which gives a new numerator and denominator; but as the diviſion can go no farther, the two denominators muſt be multiplied for a new denominator, and the two numerators, in like manner, for a new numerator, and the product will be a ſemidiapente, or imperfect 5th. Thus, $1\ \frac{\overset{2}{6}}{5} + \frac{32}{\underset{9}{27}} = \frac{64}{45}$.

VIII.

VIII. If, after the divifion is finifhed, there remain two quotients above, and but one below, or the contrary, the two quotients muft be multiplied by each other, and placed as a fraction with the one: thus, if a tone major be added to a femitone major, $1\ \frac{\overset{3}{9}}{\underset{1}{8}} + \frac{\overset{2}{16}}{\underset{5}{15}} = \frac{6}{5}$, we have a minor third for the product.

IX. In adding intervals together, the numerator of one and the denominator of the other, that are alike, are to be cancelled; or, which is the fame thing, we muft write unity over and under each, and then proceed with the divifion as before.

Thus $1\ \frac{9}{8} + \frac{16}{15} + \frac{10}{\underset{1}{9}} + \frac{\overset{1}{9}}{8}$; then $1\ \frac{9}{8} + \frac{16}{\underset{3}{15}} + \frac{\overset{2}{10}}{\underset{1}{9}} + \frac{\overset{1}{9}}{8}$;

then $1\ \frac{\overset{3}{9}}{8} + \frac{16}{\underset{\underset{1}{3}}{15}} + \frac{\overset{2}{10}}{\underset{1}{9}} + \frac{\overset{1}{9}}{8}$; then $1\ \frac{\overset{3}{9}}{\underset{1}{8}} + \frac{\overset{2}{16}}{\underset{\underset{1}{3}}{15}} + \frac{\overset{2}{10}}{\underset{1}{9}} + \frac{\overset{1}{9}}{8}$;

then $1\ \frac{\overset{3}{9}}{\underset{1}{8}} + \frac{\overset{\overset{1}{2}}{16}}{\underset{\underset{1}{3}}{15}} + \frac{\overset{2}{10}}{\underset{1}{9}} + \frac{\overset{1}{9}}{\underset{4}{8}}$; then $1\ \frac{\overset{3}{9}}{\underset{1}{8}} + \frac{\overset{\overset{1}{2}}{16}}{\underset{\underset{1}{3}}{15}} + \frac{\overset{2}{10}}{\underset{1}{9}} + \frac{\overset{\overset{1}{1}}{9}}{\underset{\underset{2}{4}}{8}} = \frac{3}{2}$, a fifth, or diapente.

This

This laſt operation includes the whole, which, had it not been to ſhew the progreſs of the diviſion, would at firſt have been preſented in this form only.

X. If we would know the intervals which compoſe the ſyſtem of diapaſon, we muſt extract them from the harmonics of any three next terms or roots; as 1, 3, 9, or 3, 9, 27. The octave of the middle or ſecond term, or principal fundamental baſs, is to be placed firſt, and the ſucceeding ſounds to follow in the order of a ſyſtem; after which we muſt divide each with the next, by a common diviſor, or by their difference, and we ſhall have the ratio of each interval, from acute to grave; whoſe ſum, when added together, is equal to the diapaſon.

Example: from 1, 3, 9, of the firſt diagram, Rule IV.

Tetrachord.

$\frac{9}{10}$	$\frac{8}{9}$	$\frac{15}{16}$	$\frac{8}{9}$	$\frac{9}{10}$	$\frac{15}{16}$	$\frac{8}{9}$	$=$	$\frac{1}{2}$
36,	40,	45,	48,	54,	60,	64,	72.	
A	G	F	E	D	C	B	A	

We have here the ratio of 36 + 40 = $\frac{9}{10}$ a tone minor, and 40 + 45 = $\frac{8}{9}$ a tone major; and of the others, placed in fractions above, in this example.

XI. It muft be obferved, that in a defcending feries from acute to grave the fraction is formed by placing the lefs number above, but in an afcending feries the lefs number of the fraction is below; as in the following example, taken from the three firft terms of the fecond diagram, 1, 3, 9.

Tetrachord.

$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$	$= \frac{2}{1}$

24,	27,	30,	32,	36,	40,	45,	48.
C	D	E	F	G	A	B	C

In thefe two examples of diapafon the great advantage of the ratio is difcovered; for though each is compofed of three tones major, two minor, and two femitones major, yet their pofition in each is very different; and as the harmonious conftitution of the tetrachord, which is the product of the three next terms, 1, 3, 9, or 3, 9, 27, &c. is the original of thefe and all other fcales, we muft look there for their effential differences.

Thus,

Thus, in the firſt tetrachord from C to D is a tone minor, which in the ſecond is a tone major: alſo from D to E is a tone major in the firſt, and a tone minor in the ſecond. Beſides this, as the intervals in each diapaſon are the products of the ſeries 1, 3, 9, which is in the ratio of 3, their motion or ſucceſſion muſt be conformable to that ſeries, which will not permit us to move from 40 to 45, becauſe they are the products of 1, 9, and not 1, 3, or 3, 9, which regulate the motion of all the others.

From theſe obſervations we diſcover that the *geometrical* progreſſion of the fundamental baſs is the *governing principle*, and that the ſyſtem of tetrachord, which made ſo great a part in the Grecian theories, had, from its ſimplicity, a preference to the diapaſon, which, when applied to the fundamental baſs, or three terms, is incapable of proceeding as a ſyſtem from one extreme to the other, unleſs the ſyſtem is diſcontinued by an incompoſite interval of a third, as in the following example, which is the ſame deſcending or aſcending.

C	D	E	F	G	A	C	B	C.
24,	27,	30,	32,	36,	40,	48,	45,	48.
3,	9,	3,	1,	3,	1,	3,	9,	3,
C	G	C	F	C	F	C	G	C.

In this progreſſion it appears how great a ſhare the ſecond term 3, or principal generator, has in reſpect to the firſt and third terms 1 and 9.

XII. To know the relation of a root and either of its harmonics, they muſt be brought as near together as poſſible, by dividing the harmonic by 2. Thus, to find the ratio of 27. 9; as I cannot divide 27 by 2, I muſt multiply 9 by 2, which gives 18, and then (by Rule IV.) $\frac{27}{18} = \frac{3}{2}$, alſo of 49. 9; 9 muſt be multiplied till brought the neareſt to 45, as $\frac{45}{36} = \frac{5}{4}$, a major third.

But in order to ſave the trouble of multiplying or dividing the terms to be compared, we may extract them from the harmonic tables.

XIII. Thus

XIII. Thus if we compare the root 3 and 729, in the columns parallel with 3, we look for one of its multiples that comes the neareſt to 729, and find it to be 768; then placing them as fractions, and proceeding by Rule IV. $\frac{768}{729} = \frac{256}{243}$, we have a limma, or the difference between C in a natural ſcale, and B in a ſcale with four ſharps. Alſo if 81, a fifth to root 27, be compared to 5, a major third of root 1, we proceed as before, except in the diviſion, and find the ratio $\frac{81}{80}$ a comma, which is the difference of A in C ſcale, and A in G ſcale; and if we add the comma to the limma, by Rule V, we have a ſemitone major: thus $\frac{256}{243} + \frac{81}{80} = \frac{16}{15}$.

In the ſame manner we may proceed to diſcover by theſe tables all other intervals, however remote: as for inſtance, the *Pythagorean* comma 524288, and 531441, which is a compariſon of a multiple of the firſt root, with the fifth of the twelfth power, root 177147.

It muſt be particularly obſerved, that every ſcale is perfected by the harmonics of the three

next

next terms, and that the product of no other can enter into its composition: thus, 1, 3, 9, or 3, 9, 27, &c.

XIV. If we would know the difference between the first and last note of a series, we must observe that the ratio is formed so as to distinguish the ascending or descending intervals, by placing the greater or less number of the fraction uppermost; agreeably to Rule XI; after which we may proceed to add them together, according to Rule IX.

C G A E F C G

Thus, $1\ \frac{3}{4} + \frac{10}{9} + \frac{3}{4} + \frac{16}{15} + \frac{3}{4} + \frac{3}{2} = \frac{3}{4}.$

In this example the last note is a fourth lower than the first.

These rules being sufficient for the *addition* of intervals, I proceed to explain how a less interval may be *substracted* from a greater, in order to discover the difference.

XV. In comparing one interval with another they must be placed as fractions, and the sign of multiplication

multiplication put between them, to ſhew that the numerator of the firſt is to be multiplied by the denominator of the ſecond; and the denominator of the firſt by the numerator of the ſecond. Thus the difference of a tone major when compared with a tone minor is a comma: $\frac{9}{8} \times \frac{10}{9} = \frac{81}{80}$, a comma. between an octave and fifth, $\frac{2}{1} \times \frac{3}{2} = \frac{4}{3}$, a fourth; between a fifth and fourth, $\frac{3}{2} \times \frac{4}{3} = \frac{9}{8}$, a tone major; between a fifth and ſixth major, $\frac{3}{2} \times \frac{5}{3} = \frac{10}{9}$, a tone minor; between a ſixth minor and a fifth, $\frac{8}{5} \times \frac{3}{2} = \frac{16}{15}$, a ſemitone major.

If the products are not in their loweſt denomination, after this croſs multiplication of the ratios is finiſhed, they muſt be divided by a common diviſor. Thus, if from a fifth we ſubſtract a major third, the remainder is a minor third: $\frac{3}{2} \times \frac{5}{4} = \frac{12}{10} = \frac{6}{5}$, a minor third.

If we ſubſtract a ſemitone major from a tone minor, we have a ſemitone minor: $\frac{10}{9} \times \frac{16}{15} = \frac{150}{144} = \frac{25}{24}$. The common diviſor here is 6, the difference between

tween 150 and 144, (fee Rule IV.) and the product is a femitone minor.

If from a minor third a deficient third is taken, the refidue is a comma: $\frac{6}{5} \times \frac{32}{27} = \frac{162}{160} = \frac{81}{80}$, a comma.

XVII. When the multiplication of the ratios is finifhed, and the products have no common divifor, we may be fure they are in their loweft denomination: as for example, the difference between the tritone 45. 32, and the femi-diapente 64. 45, $\left(\frac{45}{32} \times \frac{64}{45}\right)$ is $= \frac{2048}{2025}$, their loweft denomination.

XVIII. If the ratio, or difference, is not well underftood, we muft look for the numbers in the harmonic tables in either of the diagrams: as for example, 2048 is a multiple of root 1 B, in the firft, or of F, root 1, in the fecond diagram; and 2025 is C♭, a minor third to the fundamental bafs, root 405, in the quintuple of the firft, or is E♯, a major third to the fundamental bafs at root 405, in the quintuple of the fecond, diagram. If, after this, a more fatisfactory knowledge of this ratio is wanted, we may add it to fome other fmall quantity

tity as at Rule V. For example, $\frac{2048}{2025} + \frac{81}{80} = \frac{128}{125}$, an enharmonic diesis.

Also a semitone minor added, $\frac{2048}{2025} + \frac{25}{24} = \frac{256}{243}$, a limma. Or a greater semitone minor added, $\frac{2048}{2025} + \frac{135}{128} = \frac{16}{15}$, a semitone major.

To account for these two species of minor semitones, as well as the two species of major, we must substract either a semitone major, or a semitone minor, from a tone major, and the residue will be in the first case a greater semitone minor, and in the second a greater semitone major.

For example, $\frac{9}{8} \times \frac{16}{15} = \frac{135}{128}$ exceeds $\frac{25}{24}$ by a comma; and $\frac{9}{8} \times \frac{25}{24} = \frac{27}{25}$ exceeds $\frac{16}{15}$ by a comma.

From these divisions of the tone major, the impossibility of an *equal* division of intervals is very evident; for if we divide the tone minor, it will be also into unequal parts. Example, $\frac{10}{9} \times \frac{16}{15} = \frac{25}{24}$, a semitone minor. And the same inequality will be found in dividing all other intervals discovered in a theory of harmonics.

There are other methods by which the addition and ſubſtraction of intervals might have been explained; but as theſe appeared to be the leaſt difficult, I have therefore given them a preference, and doubt not but they will be found ſufficient when applied to the harmonic tables; in which, every interval, however minute, may be diſcovered, as alſo its root, or fundamental baſs, together with its ſcale and relation to all others, whether with minor or major thirds; for I muſt obſerve, that there are as many poſitions of theſe two ſpecies of ſcales, as there are flat or ſharp dieſes, excluſive of the two natural ſcales.

Theſe are the principal *Rules* in reſpect to the Ratio, by which the following theory will be underſtood without much difficulty.

THE

THE

THEORY

OF

HARMONICS.

PART THE SECOND.

SECTION I.

AMONG the works of the Grecians, publiſhed by M. Meibomius, we have an Introduction to Muſic by Guadentius, the Philoſopher, a favourer of the Ratio. In the 17th page of his Introduction, he has laid down *two Diagrams*, which, being in their degrees compoſed of tones major and limmas, have given occaſion to the moſt formidable objection to the Grecian Theory.

These two diagrams I come now to examine. They are to be confidered as *one general and univerfal fyftem*, in which all particular fyftems are contained.

A particular fyftem may be limited to fome part of an univerfal one, but is always a regular collection, or compofition, of many things; by which a chain of principles is linked together, and their feveral parts follow, and depend on each other.

In explaining thefe diagrams there are two things to be kept conftantly in view: the one, the different *lengths* of ftrings, which is the foundation of the firft diagram; and the other, their different *vibrations*, which is the foundation of the fecond.

Thefe diagrams, as they now ftand, are compofed of tones major and limmas, which are placed before the large numbers in the firft, and after them in the fecond, diagram. See Pl. XVIII. The fifteen names are common to each; but the numbers placed before the names reprefent the lengths of ftrings in the firft diagram, and thofe placed after reprefent the vibrations in the fecond.

But in order to difcover the original feries of which thefe numbers are compofed, they muft be divided by 2, to bring them to their loweft denomination.

$\frac{9}{8}$ $\frac{9}{8}$ $\frac{256}{243}$ $\frac{9}{8}$ $\frac{256}{243}$ $\frac{9}{8}$

2592	2304	2048	1944	1728	1536	1458
1296	1152	1024	972	864	768	729
648	576	512	486	432	384	
324	288	256	243	216	192	
162	144	128		108	96	
81	72	64		54	48	
	36	32		27	24	
	18	16			12	
	9	8			6	
		4			3	
		2				
		1				

From theſe reduced numbers a geometrical progreſſion is formed in the ratio of 3; as 1, 3, 9, 27, 81, &c. in the harmonic tables belonging to each diagram. Pl. XIII and XIV.

I ſhall now place this ſeries in the following order, beginning with unity, and annex to each number one of the ſeven muſical letters; the choice of which is determined by the greateſt number of fifths aſcending and deſcending without being obliged to make uſe of either flats or ſharps.

First Diagram. Lengths of ſtrings deſcending from unity.

B	E	A	D	G	C	F
1	3	9	27	81	243	729
F	C	G	D	A	E	B

Second Diagram. Vibrations of ſtrings aſcending from unity.

From

From this deſcription we may obſerve, that the different lengths of ſtrings are inverſe, or reciprocal, as their different vibrations; that by theſe properties in muſical ſtrings, the ſeries in the firſt diagram muſt deſcend in the ratio of 3, by a neceſſity equal with that which obliges the ſecond to aſcend from unity in the ſame ratio.

We alſo diſcover, that inſtead of a ſcale of very untunable degrees, ſuch as are tones major and limmas, we have a continued ſeries of triples, in geometric proportion, each of which being multiplied by an arithmetic ſeries 1, 2, 3, 4, &c. generates its own harmonics and all other conſonant intervals, and muſt therefore be conſidered as the root, or principal, by which thoſe harmonics exiſt, as may be ſeen in the harmonic tables, Pl. XIV, &c.

This progreſſion from each to the next is in a compound ratio of a fifth, viz. a twelfth; by which means we have the octave to 1, in our way to the fifth, 1, 2, 3. This octave appears neceſſary to every root; not only to approach the fifth in its true ratio 2. 3, but alſo to unite the two great principles on which this theory ſo much depends, which are the *geometric* with the *arithmetic* progreſſion.

Theſe octaves neceſſarily multiply the firſt ſeries of ſeven roots into fourteen, when ranged in a ſcale, as in the two diagrams, Pl. XVIII; but as the Grecians deemed this an imperfect ſyſtem, they added another ſound, viz. Proſlambanomenos, in the ratio of 1. 4, by which the whole was comprehended in two octaves, when formed into degrees, which they

they reckoned the moſt perfect ſyſtem, as containtaing all the leſs. See Pl. XIX; in which the two diagrams are inverted, and the muſical letters applied to each ſound.

That a ſeries, in the ratio of 3, ſhould be formed into a ſcale of degrees, conſiſting of tones major and limmas, ſeems neceſſary to preſerve the numbers according to their magnitude, as they repreſent the different lengths of ſtrings and their vibrations; to deſcribe both of which properly required the inverſion of the ſcales to be placed in oppoſition to each other, in order to ſhew that the ſame number which repreſents an acute ſound in the firſt, was the ſame which repreſented a grave ſound in the ſecond. But as they could not in this form be applied to either of the genders, it cannot be doubted but that ſome other form was neceſſarily implied, and generally known to be the only rational foundation of a theory of harmonics.

Neverthelеſs it muſt be owned, that for want of a more particular explanation of this doctrine, the writers of ſucceeding ages have been miſtaken, and have formed only one ſcale of degrees out of the two diagrams; and ſome writers on this ſubject have explained grave ſounds as high, and acute as low.

The whole theory thus diſordered, and every principle of ſcience extinguiſhed, gave occaſion to the great objection to the Grecians' having any knowledge of harmony; namely, that the imperfect concords, $\frac{5}{4}$, $\frac{6}{5}$, $\frac{5}{3}$, and $\frac{8}{5}$, which are ſo eſſential to harmony, are not to be found in their true ratio.

This

This objection must remain unanswerable, if no other application than that of a scale of untunable degrees can be discovered. But the contrary of this has already appeared, by reducing the large numbers to their lowest denomination, by which we have found their real application in a geometric progression in the ratio of 3.

If we examine the two diagrams, (Pl. XVIII.) the Proslambanomenos 2304 in the first diagram, is the lowest or gravest sound; but the numbers of the Proslambanomenos of the second diagram, as the gravest sound, are 648. Also the highest, or Nete hyperbolæon, of the first is 576, whereas the highest of the second is 2592. In the same manner (Pl. XIX.) the numbers in the first, to which A or B are annexed, are the same as those of G or F in the second, as D is the only note to which the same numbers are applied, and is of the root 27 in each, by being the same distance from unity in each.

The inversion of the scales, agreeably to the two great principles, makes no alteration among the intervals, which are the same above and below, as appears by the ratios, and by the black notes, which distinguish the limma, as the open notes do the tones major.

These large numbers in the diagrams have already been discovered to be a geometric progression ascending and descending, in the ratio of 3, as 1, 3, 9, &c. and have each been distinguished as a root, when multiplied by the arithmetic series, 1, 2, 3, 4, 5, &c. by which operation we discover all possible consonance belonging to that root, which cannot be applied to any

any other. As theſe harmonics are all in their true ratio to the root, as well as among themſelves, their relation and agreement with the harmonics of any other root, may very eaſily be diſcovered by the rules already laid down for the addition or ſubſtraction of ratios.

It appears in the harmonic tables, Pl. XIV, &c. that the product of each root is comprized in three letters: to diſcover therefore which of theſe three letters is the fundamental baſs, we muſt bring them the neareſt that is poſſible to each other, and the graveſt or loweſt of the three will be the fundamental baſs.

Thus in the firſt diagram of the harmonic tables, 6, 5, 4, are the neareſt to each other, and their relations ſtand thus $\frac{5}{6} + \frac{4}{5} = \frac{2}{3}$, where 6 is the grave ſound, and conſequently the fundamental baſs.

But the contrary of this is true in the tables of the ſecond diagram, where 4, 5, 6, place 4 as the fundamental baſs, as $\frac{5}{4} + \frac{6}{5} = \frac{3}{2}$.

Hence we diſcover that no root in the firſt diagram can be a fundamental baſs, as they all are in the ſecond; for inſtance, in the firſt, unity, or B, is the root, yet E 6 is the fundamental baſs; but in the ſecond, unity, or F, is the root, and alſo the fundamental baſs, whether it ſtands as 1, 2, or 4; for theſe octaves make no difference with reſpect to its power as a fundamental baſs.

It muſt be obſerved, that theſe differences are occaſioned by the two firſt principles of the lengths of ſtrings, and their vibrations; the firſt places unity in the acute, and deſcends to the grave, as the numbers increaſe; and the other places unity in the grave, and in like manner as the numbers increaſe aſcends to the acute.—To expreſs which, it was neceſſary to inſert the ratios, and place the largeſt number of the firſt contrary to what it is in the ſecond example.

This combination 6, 5, 4, or 4, 5, 6, forms the trias harmonica, which being ſung together, produces the moſt perfect union of conſonance by their mutual agreement.

But that the roots, the fundamental baſſes, and their harmonics may be well underſtood, by thoſe not perfectly acquainted with a theory formed on theſe principles, Pl. xx and xxi are deſigned to illuſtrate them.

Plate xx, comprehends the ſeven roots of the firſt diagram, in their loweſt denomination, from unity to 729, which are placed in the firſt column, 1, 3, 9, &c. deſcending from acute to grave in the ratio of 3.

The three letters in the next column expreſs the trias harmonica, of which the graveſt is the fundamental baſs, before explained.

In the third column are the numbers, or products of the roots when multiplied by 6, 5, 4, by which a compariſon of the harmonics of one root is eaſily made with thoſe of any other, by by the rules before explained for the addition, or subtraction, of intervals.

In the following columns are the multiples of the root by 2, in large numbers, which correspond with those in the first diagram, at Pl. XVIII.

Pl. XXI, containing the second diagram, has the same number of columns as the first, and for the same reasons; except instead of descending from acute to grave, it ascends from grave to acute, and the root in this is the fundamental bass, contrary to what it was in the other; and as the progression advances from 27 to 729, the series of sharps is discovered in their proper order; F ♯, C ♯, G ♯, D ♯: as the flats, B ♭, E ♭, A ♭, D ♭, are in the first diagram, from 27 to 729. We have also the harmonics for the minor mode or scale in the first diagram, and for the major in the second.

From what has been explained, it appears, that the principles on which this theory is founded are divided into two parts, and that each is of equal importance to the whole; that the ratio of concord, or discord, in the first is the same in the second diagram, but in a contrary direction from unity; that the *minor* scale is not formed *out* of the *major*, but each is *original*, and depends on distinct principles; each has its roots, its fundamental basses, and harmonics, and whatever else is necessary to compleet the most perfect theory, by supplying every demand that *Melody* or *Harmony* can make.

SECTION II.

OF THE TETRACHORD.

The Tetrachord, in the Diatonic gender, is a ſyſtem the moſt natural, and agreeable to the voice of man. Its compoſition conſiſts of four ſounds, or three intervals, of which the ſemitone is the firſt, and is ſucceeded by the tone minor, and finiſhes with the tone major, in the firſt diagram.

Theſe three intervals, being the product of any three next terms, in a geometric progreſſion in the ratio of 3, are fixed and unalterable in the diagram to which they belong; and cannot therefore be conſidered as a ſimple melody only, but as a compoſition of ſeveral, whoſe union with, and dependence on the three terms, or fundamental baſſes, makes the whole truly harmonical, as appears by the two tetrachords formed in Pl. XXII, fig. 1 and 2, extracted from the two diagrams.

In the firſt, we have all the ſounds proper for the minor ſcale, and in the ſecond the major is completed.

The loweſt ſpace of each tetrachord has the roots, 1, 3, 9, with the fundamental baſſes in the ſpace above, after which, the three next ſpaces contain the three melodies, with figures annexed to each, to ſhew their relation to each other, and to the fundamental baſs. The upper of the three ſpaces is the tetrachord, B, C, D, E, moving from left to right, with the ratio of each interval placed above, as determined by the roots, or fundamental baſſes; and the whole makes a moſt agreeable ſucceſſion

ſucceſſion of harmonic ſounds, either aſcending or deſcending, and the different poſitions of tones, major and minor, are diſtinguiſhed according to the ſcale; thus, from C to D, which is a tone minor in the firſt, is a tone major in the ſecond; alſo from D to E, which is a tone major in the firſt, is a tone minor in the ſecond tetrachord.

It follows next to examine the ſeveral parts of this harmonic compoſition of tetrachord, to ſee if any thing characteriſtic can be diſcovered in the motion of theſe harmonics, as they appear to have an abſolute dependence on the change of the fundamental baſs.—I ſhall make ſome obſervations on fig. 1, which, as there is very little difference between this and fig. 2, may be applied equally to both.

When the fundamental baſs moves from 1 to 3, the upper melody riſes a ſemitone major $\frac{16}{15}$; the ſecond melody a tone minor; and the third is a ligature, or continues in the ſame tenſion.

The next movement of the fundamental baſs is from 3 to 9, which raiſes the upper part, or melody, a tone minor higher; the ſecond is a ligature; and the third part, or melody, aſcends a ſemitone major.

The third and laſt movement is from 9 to 3, which obliges the upper melody to aſcend a tone major; the ſecond is a ligature; and the third melody deſcends a ſemitone major.

Thus we diſcover three things in this compoſition of the tetrachord.

The

The firſt is, that when the baſs moves from 1 to 3, or from 3 to 9, there are only two ſounds that move among the harmonics, and the third ſound is a ligature, by being common to both.

Of theſe two ſounds that move, one is either a tone minor $\frac{10}{9}$, or major $\frac{9}{8}$; and the other is conſtantly a ſemitone major $\frac{16}{15}$.

Now as the ſemitone major and the ligature are found to attend every change of the baſs, they are without doubt a diſtinguiſhing part of the character of the diatonic gender, and will, as ſuch, be conſtantly found among the harmonics of the fundamental baſſes in the ratio of 3.

In each tetrachord the ſemitone is marked with a horizontal line, and the ligature with a ſemi-circle or bind.

The next obſervation to be made is, that no tone major $\frac{9}{8}$ is found among the melodies, till the fundamental baſs moves from 9 to 3, and then that interval appears, and completes the formation of the tetrachord in the upper melody; for as there are but three ſpecies of intervals in the diatonic ſyſtem, it ſeems neceſſary for the principal melody to be compoſed of them all, in order to eſtabliſh a ſyſtem, founded in nature, on the moſt certain and obvious principles.

The third obſervation is, that the tetrachord is found to be a particular melody, generated by the fundamental baſs, and ſubject to harmonic powers, and muſt, therefore, be conſidered

ſidered in a ſecondary relation with reſpect to firſt principles, directly contrary to the method obſerved by all Theoriſts, who have made it a firſt and original one, by miſtaking the effects for the cauſe, without any regard to minor or major ſcales: by which means they have confined their obſervations to a ſimple melody, contracting a ſyſtem founded on harmonic principles to three ſimple intervals, without conſidering that the ſcale or ſyſtem of the diatonic octave was ſo blended and united with the tetrachord, that all its intervals were determined by the ſame three terms, which formed the harmonious conſtitution of the tetrachord.

To illuſtrate this truth, I have at fig. 3 and 4, Pl. XXII, ſelected from among the harmonics of 1, 3, 9, of the tetrachords, fig. 1 and 2, the ſounds proper for a ſcale of diatonic degrees, which are ranged and diſpoſed agreeably to the order of minor and major ſcales, together with the roots or fundamental baſſes, to which they reſpectively belong. But as they ſtand oppoſite to each other, we more readily diſcover the difference between intervals formed for melody in the diatonic, and thoſe proper for a baſs; or, as the Grecians have diſtinguiſhed, between ſyſtematic and diaſtematic intervals.

To underſtand the compoſition of theſe ſcales, it muſt be obſerved, that the roots, 1, 3, 9, are placed in the firſt or loweſt ſpace; the fundamental baſſes are in the ſecond, expreſſed by the multiples of the roots, the more eaſily to diſcover the interval formed between the ſyſtem or ſcale, and the baſs, which is ſet down as a ratio in the third ſpace. The fourth ſpace contains the numbers, which are to be compared with

with thoſe of the fundamental baſſes; and the fifth ſpace contains the letters which thoſe numbers repreſent, together with the ratio of the interval, from any letter to the next.

This ratio is diſcovered by comparing one number with the next in the fourth ſpace; thus, $\frac{36}{40} = \frac{9}{10}$, and the like. The ratio, or interval between the baſs and the melody, is diſcovered by comparing the numbers in the ſecond ſpace, with thoſe of the fourth; thus, $\frac{35}{72} = \frac{1}{2}$ an octave.

I have been particular in explaining theſe ſcales, becauſe by the ſame method the tetrachord and its harmonics, and the ſcale which is included in that ſyſtem, may be formed from any three next terms in the harmonic tables, as 3, 9, 27, or 9, 27, 81: obſerving, however, that in order to bring the numbers neareſt to each other, we may multiply or divide by 2 any number in the tetrachord for that purpoſe; thus, in the preſent caſe, in the tetrachord fig. 1, C 30 is an octave different from C in the ſcale fig. 3, becauſe 60, 64, is nearer each other than 30, 64: alſo, at fig. 2, E 60, is not ſo near as 30 is to 32; the firſt being a compound, and the laſt a ſimple ratio.

I ſhall now examine the ſyſtem of diapaſon or octave, as it has had a preference to the tetrachord by ſome modern theoriſts.

In the firſt ſcale, fig. 3, Pl. XXII, the fundamental baſs moves from E 48, to D 54; or, which is the ſame, from 1 to 9. Alſo in the ſecond ſcale, fig. 4, from F 32, to G 36; or, or, as before, from 1 to 9, which in its neareſt degree is a tone major, $\frac{8}{9}$.

Thus the original character of the fundamental baſs, which had its motion from a geometric progreſſion in the ratio of 3, is loſt; and the diatonic is ſubſtituted in its place as a baſs, contrary to every principle of harmonics.

From theſe obſervations we diſcover the impoſſibility of forming a theory of harmonics from the ſyſtem of diapaſon; for it appears, that this ſeries is not capable of continuing a ſucceſſion from one extreme to the other, as there is no communication in either ſcale between 40 and 45; or, which is the ſame, between 1 and 9, and the utmoſt that can be made of the ſyſtem of diapaſon has already been explained in page 122. (Rule XI.) In which an incompoſite interval preſerves the geometric progreſſion of the baſs, by the interpoſition of 48, between 40 and 45; after which, the remaining part of the ſeries is perfectly regular, and conformable to the three terms in the ratio of 3.

Thus the perfection of the tetrachord, and the reputation of thoſe, whoſe theories for many ages have been regulated by its harmonic powers, muſt be acknowledged; but as this is not the only inſtance in which it is found ſuperior to the harmonic ſyſtem of diapaſon, or to any other ſyſtem, I ſhall proceed to the conjunction and diſjunction of the tetrachords.

SECTION III.

OF THE CONJUNCTION AND DISJUNCTION OF TETRACHORDS.

This diſtinction of *Conjunction* and *Disjunction* will beſt be underſtood by conſidering, that as a tetrachord is the product of any three next terms in the geometric progreſſion, it follows that any new term added muſt unavoidably make ſome alteration among the harmonics; in conſequence of which, a new ſcale will be formed. For this reaſon, it is abſolutely neceſſary to fix and determine theſe alterations by principles the moſt certain.

For this purpoſe I ſhall take three tetrachords, and place them as antecedent, mean, and conſequent, in order to compare the harmonics of one tetrachord with thoſe of another, to diſcover their difference.

Thus they will ſtand as at Pl. XXIII and XXIV, and the progreſſion will be 1, 3, 9, 27, 81.

(Mean: 3, 9, 27; Antecedent: 1, 3, 9; Conſequent: 9, 27, 81.)

I ſhall begin with the mean, 3, 9, 27, and compare it with the conſequent 9, 27, 81, to which it is conjoined: for it muſt be obſerved, that the mean has no other motion, than to the antecedent, or conſequent.

As it is indifferent which diagram is taken for this compariſon, the ratios being the ſame in both, I ſhall take thoſe at Pl. XXIV of the ſecond diagram, fig. 2.

In

In comparing C, root 3, with the major 3d, C ♯, root 81, a ſemitone minor of the greateſt magnitude is found in the ratio of $\frac{135}{128}$, inſtead of $\frac{25}{24}$. Alſo the 3d to root 3, is a comma different from the 5th of root 81. (See the harmonic tables.) As the ſame irrational intervals exiſt in comparing the mean tetrachord with the antecedent, they need not be repeated; but we may proceed to the antecedent and conſequent. In comparing a multiple 256 of root 1 F, with 243 E, which is the fifth to root 81, we have the limma of the ancients; alſo in comparing 32, a multiple of root 1, with root 27, we have a 3d deficient by a comma $\frac{32}{27}$.

Beſides theſe defective intervals, others may be found, if theſe were not ſufficient, to ſhew the impoſſibility of their exiſting in one and the ſame ſcale; and therefore they cannot affect the compoſition of the tetrachord, which has been found extenſive in its harmonic powers, and perfect in all its parts. Inſtead, therefore, of objecting to theſe intervals, which are only to be found in comparing the compoſition of one perfect ſyſtem with another, they are ſo many proofs of the perfection and regularity of this theory, in which no new term can be added to the firſt three, without giving immediate information of the change of the ſcale, by the alteration of one, or more ratios; for as long as the geometrical progreſſion is obſerved in the roots, ſo long each ſcale will be perfect: and if we conſider the various revolutions among the harmonics, which muſt unavoidably happen between 80 and 81; or, 128 and 135; or, between 243 and 256; it will be impoſſible to diſcover any interval among any three next terms, that is not the moſt perfect, as well as neceſſary for the mutations of the ſcales; for as the ſucceſſion of ſounds is natural and agreeable, the variety which they produce by the mutations is one of the moſt animating parts of muſical compoſition.

Thus the origin and neceſſity of theſe intervals are diſcovered, and the objections of modern theoriſts removed.

The *Disjunction* of the tetrachords has the ſame principles for its foundation, as the conjunction; viz. the motion of the fundamental baſs, in a geometric progreſſion; to preſerve which, this diſtinction is unavoidable in a theory of harmonics, but does not appear of any conſequence, when conſidered only as a melody, independent of the fundamental baſs.

In Pl. xxiv, there are three tetrachords and two diſjunctions, fig. 4.

Theſe tetrachords are formed from the ſame roots as thoſe above, fig. 2, but are diſpoſed in a different order. The firſt tetrachord begins at 9, and ends at 27; the next begins at root 3: but as there is the ſame objection to 27, 3, as there was before to 1, 9; or 8, 9; tone major, the interpoſition of another root, 9, at the disjunction becomes abſolutely neceſſary to preſerve the progreſſion in the ratio of 3, as 27, 9, 3, and not 27, 3.

The ſame obſervation is to be applied to the laſt note at 9, in the ſecond tetrachord, and the firſt of the third, root 1; where, for the ſame reaſon, 3 muſt open the communication between 9 and 1, to preſerve the progreſſion 9, 3, 1, and prevent

vent a diatonic motion in the fundamental baſſes: but, beſides this, there are two other reaſons of equal importance; the firſt is the mutations of the ſcales, which decreaſe the number of flat dieſes, as at fig. 3, Pl. XXIII, and alſo the ſharps, fig. 4, Pl. XXIV. For from two flat and two ſharp dieſes, which are neceſſary among the harmonics in the firſt tetrachord, fig. 3 and 4, we change to one in the ſecond, and proceed to the natural ſcale at the third tetrachord; but the contrary of this may be obſerved in conjoined tetrachords of each diagram; where, from the natural ſcale, the mutation was made firſt to one, and then to two dieſes. See fig. 1 and 2, Pl. XXIII and XXIV. The ſecond reaſon is, that the melody deſcends in the ſecond diagram, and aſcends in the firſt, a tone major from the laſt note of one tetrachord, to the firſt note of the next tetrachord, by disjunction; which the Greeks diſtinguiſhed by the name of the diezeutic tone, or tone of disjunction: now as this is a tone major, and is preceded by a tone major in the former tetrachord, and as two tones major can never ſucceed to each other in the ſame ſcale, the disjunction became abſolutely neceſſary, not only to form a new phraſe and a new ſcale, but alſo to prevent the diſagreeable effect of a very ſudden and unexpected change of the dieſes, in moving from root 27, to 3.

From the nature of this tone of disjunction we may acquit the Grecians of the charge, ſo often brought againſt them, of forming their diatonic gender with tones major and limmas only; ſince the harmonic explanation of their tetrachords proves the contrary.

It

It muſt alſo be obſerved, that the ligature and ſemitone, two principal characters among the different melodies of every tetrachord, as has been already explained, are formed by the motion of the fundamental baſs in the ratio of 3, and attend every ſuch motion; and that the flat and ſharp dieſes begin in each diagram at root 27, and may be continued in an increaſing ſeries, in proportion to the number of conjoined tetrachords, from one to five, or more, dieſes; the flats deſcending by fifths, and the ſharps aſcending by fifths, in this order.

B ♭,	E ♭,	A ♭,	D ♭,	G ♭,	deſcending.
1,	2,	3,	4,	5,	
F ♯,	C ♯,	G ♯,	D ♯,	A ♯.	aſcending.

But contrary to this, the disjunction of the tetrachords decreaſes the number of flat or ſharp dieſes, from 5 to 4, and 3, &c. and thus the mutations may be made from any given or principal ſcale a fifth higher, or a fifth lower, by the increaſe or decreaſe of the flat, or ſharp dieſis; or, which is the ſame, by the conjunction or disjunction of the tetrachords.

From theſe obſervations on the conjunction and disjunction of the tetrachords, the diſtinction between them appears the moſt neceſſary to ſupport a theory in all its parts, on the moſt ſatisfactory and rational principles. We have ſeen ſeveral melodies determined by the progreſſion of the fundamental baſs, from which different ſcales have been formed, and the number of dieſes peculiar to each has been limited, without a poſſibility of the leaſt alteration, unleſs a new term is introduced, which leads either to a conjunction, or a disjunction of the tetrachords, in in which the agreement between the ſcales is found the moſt perfect; and the irrational intervals, which have already been explained, are removed to too great a diſtance to give the leaſt offence. Thus in the preſent caſe, if E, D, is a tone minor in the antecedent tetrachord, fig. 2, Pl. XXIV, and a tone major in the conſequent, yet, as we muſt paſs through the mean or middle tetrachord, before ſuch alteration can take place, no diſagreeable conſequence can be diſcovered, eſpecially as the mutations are ſupported by the progreſſion of the fundamental baſs, in the ratio of 3, whoſe harmonics are always ſo perfect, as to admit of no alteration whatever, unleſs a new term be introduced; the conſequence of which is, a new ſcale will immediately be formed in the place of the former, without any imperfection in the ratio of its harmonics.

SECTION IV.

EXPLANATION OF THE QUINTUPLE PROPORTION.

The three *primes* are 2, 3, and 5; but as a progreſſion of the firſt, 2, 4, 8, &c. can yield no variety, the harmonics being the ſame in all; and as the triple progreſſion 1, 3, 9, &c. has been fully explained, there remains only the quintuple to be conſidered, by which we ſhall diſcover a *new* motion in the fundamental baſs of aſcending, or deſcending a third major or minor, which by a judicious mixture with the triple proportion, will produce a variety of intervals not yet inveſtigated, and add greatly to the powers of the mutations.

Among

Among the many diſcoveries made in this theory by the triple proportion of the fundamental baſs, no ſemitone minor has yet been found in the ratio of 25, 24.

The uſual method of diſcovering this interval has been by dividing the tone minor into two parts; or, which is the ſame thing, ſubſtracting a ſemitone major from a tone minor, $\frac{10}{9} \div \frac{16}{15} = \frac{25}{24}$, and the remainder is a ſemitone minor. (See Rule 15, p. 126.) If this interval were the only object in view, we might be content with either of theſe methods.

Some have aſſerted, that the ancients had not 5 among the number of their primes, yet at the ſame time acknowledge that the enharmonic dieſis is an interval found in their theories, which is in the ratio of 128, 125. Now as 125 is a multiple of 5, it follows that 5 muſt be one of the primes, as well as 2 and 3. The arithmetic ſeries, which has all along been applied to the roots in the triple progreſſion, will in like manner be applied to the quintuple, and will demonſtrate this truth.

The quintuple proportion is the only one now left to complete a theory of harmonics; for, beſides the ſemitone minor, the fundamental baſs will acquire a new motion of aſcending and deſcending a third, and a very conſiderable addition will be made to each of the diagrams; for the firſt, which conſiſts of ſcales with minor thirds, will be improved with thoſe of major, and the ſecond diagram, which conſiſts of ſcales with major

major thirds, will in like manner have their reſpective minor ſcales.

The quintuple progreſſion, like that of the triple, is of no conſequence in harmonics unleſs united with the arithmetic ſeries: for from the firſt to the third term in each is diſcord, as $\overparen{1, \quad 9}$, or $\overparen{5, \qquad 125}$; and therefore each as a root, muſt be multiplied by the arithmetic ſeries 4, 5, 6, or 6, 5, 4, according to the diagram to which they are to be applied. Thus $5 \times 6 = 30$, $5 \times 5 = 25$, $5 \times 4 = 20$, and the products are the harmonics to that root, 5; alſo $25 \times 6 = 150$, $25 \times 5 = 125$, $25 \times 4 = 100$, are the products, or harmonics of root 25; and if the ſame multiplication is repeated at root 125, we ſhall have the harmonics to that root, or to any other in the quintuple; but it muſt be obſerved that 1, 5, or 5, 25, are double compounded ratios, being each a ſeventeenth major; yet as F, A, or A, C♯, are major thirds without any diſtinction of compound or ſimple thirds, ſo theſe are brought the neareſt into a ſimple ratio, 4, 5, or 20, 25, which is the ſame; in like manner as 1, 3, was before brought to 2, 3; but this will not be difficult, as the letters are annexed to each number in the harmonic tables of the quintuple, which immediately ſucceed thoſe of the triples in each diagram, Pl. XV and XVI, which have been explained page 115. But as the roots in theſe tables move in the ratio of 3, as 5, 15, 45, there are two other Plates, XXV and XXVI, which are intended to diſcover the eaſy communication between the minor and major ſcales, with the ſame number of flat or ſharp dieſes in each diagram; by which the mutations from a minor to a major ſcale, or the contrary, will be readily underſtood.

Each of theſe tables is divided into four ſections. The three columns of the firſt ſection contain ſeven roots from unity to root 729, in the ratio of 3, together with the letters, and the numbers which expreſs their relation as harmonics to each root, in the ſame manner as they ſtand in the other harmonic tables.

In the ſecond diagram the 3d to the root in the firſt ſection is the root to the ſecond throughout, after being brought to its loweſt denomination. Thus, 20 is reduced to 5, the root of the ſecond ſection, which, when multiplied by the arithmetic ſeries 4, 5, 6, or 6, 5, 4, gives the harmonics of root 5, as 20, 25, 30. Alſo if 60 be taken as a root, its loweſt form is 15, which, being multiplied as before, gives 60, 75, 90, for its harmonics.

It muſt be remembered, that in the firſt diagram of the quintuple, (Pl. xxv.) the upper letter is the root, and the loweſt letter in each root is the fundamental through every ſection.

As there is no alteration in the thirds to each fundamental, the fifth to each root in the firſt ſection is applied as a third to that of the ſecond ſection; as from 24, root 1, to 20, root 5; by which means we have two ſpecies of thirds to each root, or fundamental, throughout the three laſt ſections. Thus, 24, from root 1, is applied to root 5, and 72 to root 60, or 15; in conſequence of which, we have the two ſpecies of thirds to each fundamental, and the ſemitone minor in its true ratio 24, 25; and by purſuing the ſame method with the other five roots the ſame advantages will follow.

The

The third ſection is formed from the ſecond, and the fourth from the third, in the ſame manner as the ſecond was from the firſt; that is, the major third to the root of the ſecond ſection is 25, which is the root of the third; and the third major to the root of the third ſection is in like manner carried forward, as a root to the fourth ſection 125, and the other operations follow the ſame as at firſt; and each of theſe roots is multiplied by 4, 5, 6, or 6, 5, 4.

Every three next terms in the ſame ſection, by being in the ratio of 3, form a tetrachord, and conſequently a ſcale peculiar to thoſe three terms only; and the mean or middle term is the principal ſound, or tone, to which all the other ſounds in the ſcale refer, as the generator of the whole; in the ſame manner as has before been obſerved in the triple progreſſion. There is, however, a very great difference between theſe two proportions: for any three next terms in the triple may move in progreſſion, as 1, 3, 9; but it is not ſo in the quintuple, 1, 5, 25, or 5, 25, 125.

Therefore, as the triple proportion has been found to be the original of the quintuple, and that the latter has not, like the former, a progreſſion of three terms, it follows that the motion of the fundamental baſs in the quintuple muſt return to that of the triple, immediately after the change of riſing or falling a third.

Thus in the firſt diagram, Pl. xxv, the motion from the fundamental baſs A 72, to F 90 and B♭ 270; or from A 72, to F 90 and C 30; is better than 72, 90, and D♭ 450; or from D♮ 216,

 to

to F 90 and A♭ 150; becauſe the fundamental baſs, by moving firſt a third and then one or more fifths, in the triple proportion, leaves us in no kind of ſuſpence as to the mutation of the ſcale; but a motion of two greater, or two leſs, thirds in ſucceſſion throws all the obſcurity that is poſſible on the mutation, and leaves us in a painful uncertainty, till it is ſucceeded by the triple proportion to determine the ſcale.

The ſame obſervations may be made on the ſecond diagram, Pl. XXVI, where the motion of the fundamental baſs from C 48, to E 60 and A 20, or from 48, to 60 and B 180, is very good, and leaves us in no doubt with reſpect to the ſcale; but it is not ſo from 48, to 60 and G♯ 300, in which the motion of two major thirds in ſucceſſion in the baſs makes ſo great an alteration in the ratios of the intervals, as cannot be admitted, unleſs on ſome very extraordinary occaſion, to ſupport a particular ſentiment, in which reaſon has little to-do in the government of the paſſions.

The motion from F 16 to C♯ 100; or, which is the ſame, from F, root 1, to C♯ 25, or from A, root 5, to E♯, root 125, is equally exceptionable with the former; becauſe in this caſe the fundamental baſs moves an interval, conſiſting of two major thirds, in the ratio of $\frac{25}{16}$.

As it ſeems contrary to the quintuple proportion to move a fundamental baſs a minor third, it is therefore neceſſary before we proceed to explain the nature of this motion.

The

The change of harmonics in the triple proportion, as from 1 to 3, or from 3 to 9, has been always attended with a ligature, or one found common to both roots; agreeably to the rule which Euclid (p. 21) has laid down to be obferved in the mutations.

This will appear at Pl. XXV, in which the fundamental bafs of root 9, which is D 216, may move a minor third to F 90, which, in the quintuple, is the product of the bafs A 72 at root 3; for in this change we have one, or, if we pleafe, two founds that are common. But if A 72, at root 3, fhould move to B♭ 270, the product of root 9, we have no ligature or common found attending the change; and therefore it appears that there is no communication between 72 and 270, without firft moving to F 90, the product of 72.

Hence we have a pofitive law, which forbids a fundamental bafs to move a femitone, in like manner as before to move a tone major 9, 8, in the triple progreffion.

The fame rules muft be obferved in the fecond diagram, Pl. XXVI; where G 144 may move to E 60; but C 48 cannot move to B 180, for the reafon before given, not having a common found.

Thus it appears that the motion of a minor third in the fundamental bafs has its original in the quintuple proportion, as well as the major third; and the laws which govern one, are equally applicable to the other.

Thefe

These and the preceeding tables of the firſt and ſecond diagram contain all the various ſpecies of intervals, both rational and irrational; for by uniting the arithmetic with the triple proportion, and ſometimes with the quintuple, the ſcale or tone is perfected, as well as the mutations proper for every mode.

We have alſo not only two ſcales with minor and major thirds, peculiar to one or more flat or ſharp dieſes, but the two natural ſcales with different ſpecies of thirds.

Each ſcale is the product of any three next terms; and the ſecond, or mean term, is the tone or principal ſound of that ſcale. The relative ſcales, connected with the principal one, are regulated by the increaſe or decreaſe of one dieſis more or leſs than what completed the firſt propoſed ſcale; whereby three minor and three major ſcales may be uſed in every mode. But theſe five auxiliary ſcales make no alteration in the name of the mode, which is notwithſtanding diſtinguiſhed by that of the principal tone of the ſcale firſt taken. Thus if we take any three next terms, as G 9, D 27, A 81, which may repreſent as many principal tones of as many different ſcales, the ſecond or mean term 27 is that which governs the whole, and gives its name to the mode. And agreeably to what was obſerved before of the increaſe or decreaſe of the numbers of dieſes, 27 has two, 81 has three, and 9 has only one dieſis; and as the formation of two ſcales of a different kind depends on the ſame number of dieſes, we have ſix ſcales in the whole, which complete the mode and all its mutations

But

But as an example on this ſubject will better be underſtood, I have, at Pl. XXVII, extracted from the firſt diagram, Pl. XXV, three roots proper to form three ſcales, as at fig. 1, in which the principal tone of each is in the proportion of 9, 27, or 81, with a minor third to each; and if conſidered as a mode, the ſcale which governs it is the ſecond or mean term 27, which, to be perfect, requires two dieſes, with three at root 81, and one only at root 9.

Oppoſite to theſe, at fig. 2, are the roots of three more ſcales, in the ſame proportion of 9, 27, 81, notwithſtanding they are expreſſed by large numbers.

Each of theſe has major thirds, and the ſame number of dieſes with thoſe which are parallel with them, to form their different ſcales. Thus G 27, a minor third, with two dieſes, and B♭ 270, with two dieſes a major third, correſpond in like manner with each other, as does D 9 with F 90, or C 81 with E♭ 810.

The pitch of the principal tone of the mode may be at G 27, with a minor third, or at B♭ 270, with a major third, and the other five ſcales are the auxiliaries; and the whole being regulated by the number of dieſes, forms one mode only, according to the doctrine of the ancients.

From this explanation of the geometric proportion in the ratio of 3, it appears to be an *univerſal leading principle* in a theory of harmonics, and extends its powers through every eſſential part.

Thus

Thus the three terms of every ſcale or tetrachord, the three firſt tetrachords of the Grecian diagram conjoined, or, which is the ſame thing, the three principal ſcales in the ſame kind of every mode, are formed by theſe proportions. Beſides which, the three hexachords of Guido, the moll, the natural, and the quadro, together with the three cliffs of the moderns, F, C, and G, the order alſo and ſucceſſion of the flat and ſharp dieſes are governed and regulated by the ſame geometric proportion in the ratio of 3; and as the two Grecian diagrams have already been found throughout in the ſame ratio, there can be no doubt of the conſummate knowledge which the Greeks had of Muſic as a ſcience, and of Harmony; without which a progreſſion in the ratio of 3 could be of little or no uſe.

SECTION V.

OF THE MUTATIONS.

The Mutations (before fully explained according to the directions given by Euclid) may be reduced to the five following rules:

I. A mutation, or change from one ſcale to another, may be a fifth higher or lower; that is, from 27 to 81, or the contrary; or from 27 to 9, and the contrary. See Pl. XXVII, fig. 1 and 3.

II. A third higher or lower, which will be from the minor to the major ſcale, or the contrary.

III. A

III. A diſſonant mutation may be by riſing or falling a tone 9, 8, which is done by moving from 9 to 81, or the contrary.

IV. A mutation is made by changing the genius of a ſcale; as from D with one flat to D with two ſharps, or the contrary.

V. By a change of the mode; when the firſt movement is in one mode, and a ſecond movement ſucceeds in another, which ſometimes bears a relation to the firſt, and at other-times none at all.—Inſtances of this are to be met with in many Cantatas, Recitatives, &c.

SECTION VI.

OF DISCORDS.

The motion of the fundamental baſs having been determined by the triple and quintuple proportion, and their ſeveral concords diſcovered by the arithmetic ſeries, it remains to conſider the origin of diſcords, their different ſpecies, and how they become an eſſential part of harmonics.

The ancient writers, in treating of diſcords, have paſſed rather too ſlightly over a ſubject of ſo great importance.

Ariſtoxenus, in his diviſion of intervals, ſays, (p. 16) "The "firſt diviſion of intervals is that by which they differ from "among themſelves in magnitude."

" The ſecond, that by which the conſonant differ from " the diſſonant;" (p. 45) and yet he gives no other rule of diſſonance, or diſcord, than the difference between diapente and diateſſaron, which is a tone major 9, 8.

Euclid, in his diviſion of intervals, (which I have before taken notice of) ſays, (p. 8) that " The conſonant intervals " differ from the diſſonant" and that " the latter are placed " between the former;" and then proceeds to tell us, that " conſonance is a mixture of ſounds differing in acuteneſs and " gravity, but that diſſonance, on the contrary, refuſing to mix, " hurts the ear with harſhneſs."

From a writer of Euclid's eminence more might have been expected, eſpecially as Ariſtoxenus had ſaid ſo little on the ſubject.

Nicomachus comes nearer the point; for he tells us (p. 11, &c.) that Pythagoras, in paſſing by a ſmith's ſhop, heard different ſounds, from the ſtriking of hammers on an anvil, which engaged his attention ſo much as to put him upon trying ſome experiments, to diſcover the exact relation thoſe ſounds had to each other: and after different trials of the weights, he proceeded to try ſome experiments with ſtrings, and diſcovered the proportions of 6, 8, 9, 12; and by comparing either of the extreme with either of the mean proportions, as 6, 8, = $\frac{3}{4}$, or 6, 9, = $\frac{2}{3}$, or 12, 9, = $\frac{4}{3}$, and 12, 8, = $\frac{3}{2}$, he diſcovered the diateſſaron and diapente in their true ratios, and alſo the diapaſon from the two extremes, 6, 12, = $\frac{1}{2}$: but with reſpect to the

the mean proportions 9, 8, he found no agreement in their relation, and therefore pronounced them diſcord; and by inſerting the ſeveral degrees of ſound between 6, 8, 9, 12, he diſcovered the diatonic gender, compoſed of two tetrachords disjoined, equal to the ſyſtem of diapaſon.

What uſe Pythagoras made of this diſcovery we are not informed, for Nicomachus, who wrote this little manuel in a hurry to oblige a lady, excuſes himſelf from being more particular by the ſhortneſs of the time, but promiſes to treat every part more at large in his Commentaries; however as no ſuch book has come down to us, we muſt be ſatisfied with what is now before us.

This account of Pythagoras has been objected to by many writers; but as the numbers and their ratio correſpond with the different lengths of ſtrings and their vibrations, it is ſufficiently ſatisfactory on the preſent occaſion.

In examining theſe proportions 6, 8, 9, 12, we find,

1. That by reducing theſe numbers to their loweſt denomination we diſcover the three roots or fundamental baſſes, 1, 3, 9. See the two firſt columns in Pl. XXVIII and XXIX.

2. That by applying the arithmetic ſeries to each of theſe three terms, the ſyſtem of the tetrachord and the ſcale have already been formed in each diagram. Pl. XXII.

3. That the character of a principal ſound or generator has been diſcovered in the mean or middle term, by the application

of the two firſt original principles of the different lengths and their vibrations. See Pl. XII; in which, the fifth above and fifth below unity are in the ratio of 3, which anſwers to the ſeries, 1, 3, 9.

4. The fifth, being thus generated by either of theſe principles, diſcovers its union, as a principal harmonic ſound with every fundamental baſs, whether it has a conſonant or a diſſonant combination applied to it.

5. It has before been obſerved that the two products of 3, a generator, 1, 9, or 9, 1, cannot *ſucceed* to each other as fundamental baſſes; but this does not prevent their being *united* in harmonic combination; eſpecially if by this method the different ſpecies of diſcords are formed, agreeably to the obſervations made by Ariſtoxenus, Euclid, and Pythagoras.

The firſt ſays, that "a diſcord is the difference between diateſſaron and diapente;" which correſponds with the 9, 8, of Pythagoras; and Euclid ſays, "that the diſcord is placed between the concords;" in like manner 9, 8, is placed by Pythagoras between the concords, 6, 8, 9, 12. But Euclid proceeds further, and ſays "the diſcords will not mix like the concords;" Pythagoras removes this objection, by *combining* the diſcord with other ſounds that will mix; of which the fifth, as was obſerved before, muſt always be one, as appears in this Pythagorean combination, 6, 8, 9, 12.

That this and other diſſonant combinations, diſcovered by the union of the firſt and third terms, 1, 9, or 8, 9, may be better underſtood,

underſtood, I have placed them in the two Pl. XXVIII and XXIX.

The firſt is agreable to the different lengths of ſtrings, the ſecond to their vibrations; and as the ſpecies of diſcords are the ſame in both, ſo the obſervations, which I ſhall make on one, may be applied to the other, except the difference which is found between the minor and major third; which will be taken notice of in its proper place.

The firſt column in Pl. XXIX has the three terms, or fundamental baſſes, 1, 3, 9.

The ſecond has the *Pythagorean* numbers 6, 8, 9, 12.

The third has the harmonics to the three terms.

The fourth column has a diſcord, which is the ſuſpenſion of the fourth formed on C, a fundamental baſs, to which F is a fourth, and is a diſcord to G, a fifth to the root C.

In the fifth column is another diſcord, viz. the ſuſpenſion of the ninth, formed on F, a fundamental baſs, to which A is a third major, C a fifth, G a ninth and a diſcord to the root.

The ſixth column is a third term compounded, having a minor ſeventh to G, a fundamental baſs, to which B is a major third, D a fifth, and F the diſcord is a minor ſeventh, and forms the compounded harmonics of the third term.

The

The ſeventh column is a double compound, having a ninth major or minor to G, a fundamental baſs, to which B is a major third, D a fifth, F a ſeventh, and A ♮, or ♭ a diſcord and a double compounded third term, as in the firſt diagram the ſeventh column is a double compounded fourth term. Thus, by the union of the firſt and third terms, 1, 9, or 8, 9, theſe three ſpecies of diſcords are formed, and ſupported by their different fundamental baſſes, and their fifths.

The additional harmonic powers, which theſe three terms, or fundamental baſſes, acquire by the application of the diſcords, contribute very much to diſtinguiſh their different characters, by which their ſucceſſion is eaſily diſcovered; as for example;

The firſt term from its ſituation, is incapable of receiving any other diſcord than the ſuſpenſion of the ninth.

The ſecond term may have either of the ſuſpenſions, but not the minor ſeventh.

The third term has more extenſive powers than the other two; for beſides the ſuſpenſion of the fourth, or the ninth, it can receive the minor ſeventh and major third, which is peculiar to that term only, as will be explained in page 168.

Hence we may obſerve, that when Pythagoras determined 8, 9, to be diſcord, he could not, as many have imagined, intend a mere ſucceſſion of ſounds; (for then the tetrachord and other perfect ſyſtems muſt have been a ſucceſſion of diſcords;) but on the contrary, a combination with other ſounds. For if we conſider, that all communication is cut off between the

the firft and third term, 1, 9, as fundamental baffes, it feems as if their *union* was referved to be exerted in a frefh creation of harmonics, and to give new and diftinct characters to the three terms.

Having explained the difcords in Pl. XXIX, I fhall now compare them with thofe in Pl. XXVIII.

The firft five columns are the fame, except that minor thirds are next the fundamental bafs; but the fixth column in each being compared difcovers a difference of very great confequence to them otion of the fundamental bafs.

Each contains a feventh and a difcord; the firft has been found to have a major third, but the other on the contrary has a minor third next the fundamental. Thefe difcords, when compared with their refpective fundamental baffes, are found in the fame ratio; but when compared with the other harmonic parts are very different. Thus,

Pl. XXIX, from E to D, is a minor 7th in the ratio of 16, 9.

Pl. XXVIII, from G to F, is alfo a minor 7th in the ratio of 9, 16.

Pl. XXIX, from G to D, is a 5th deficient by a comma = 40, 27.

Pl. XXVIII, B to F, is a femidiapente or imperfect 5, = 45, 64.

Pl. XXIX, from B to D, is a minor 3d deficient by a comma, = 32, 27.

Pl. XXVIII, from D to F, is alfo a minor 3d deficient by a comma, = 27, 32.

In

In comparing the ſeventh with the baſſes and their harmonics, we diſcover no other difference than that which ariſes from the two ſpecies of thirds to the fundamental baſſes; for theſe thirds, when compared with the ſevenths, give two ſpecies of fifths, which is confirmed by adding each ſpecies of third to its particular fifth, and the product is the ratio of the ſeventh; $\frac{5}{6} + \frac{27}{40} = \frac{9}{16}$; alſo $\frac{5}{4} + \frac{64}{45} = \frac{16}{9}$.

However imperfect theſe fifths, as well as the deficient thirds may appear at firſt ſight, yet as they are occaſioned by the addition of a new harmonic, to which the others are compared, they cannot be objected to, not only becauſe the whole combination is extracted from the ſcale, which admits of no ſound that is not a product of the three terms, but alſo becauſe each interval in the whole combination, when compared with the fundamental baſſes, is in its true ratio.

Hence it appears that all harmonics, whether ſimple or compound, are not to be queſtioned as to their mutual relations, provided they are true to the fundamental baſs, whoſe powers they are intended to encreaſe, by ſupporting its operations.

This will appear very evident in the diſtinct characters of theſe two ſevenths, which I ſhall explain in their turn: but as that with a major 3d, Pl. XXIX, has ſomething very intereſting in its compoſition, I ſhall begin with that firſt.

This ſeventh is compoſed of three ſpecies of thirds, from G to B, B to D, and D to F, in the following ratios; $\frac{5}{4} + \frac{6}{5} + \frac{32}{27} = \frac{16}{9}$.

The

The D, and the F, which make the deficient third by a comma, are not only original ſounds in the ſcale, but are alſo found in the geometric progreſſion, 1, 3, 9, 27, and therefore are fixed and unalterable, whether they appear as fundamental baſſes, or harmonics.

The active principle of the ſemitone major has already been obſerved in the formation of the tetrachord, Pl. VII, Fig. 2 and 4, which is conſtantly formed, when the baſs moves in the ratio of 3; and as there are two ſemitones in every ſcale, one of which attends the baſs, from 1 to 3, and from 3 to 9, or the contrary; ſo, in this compounded harmonic, now under conſideration, theſe two ſemitones move in a contrary direction, when the baſs moves from 9 to 3. Thus the major third to 9 aſcends a ſemitone major to the octave 3, the ſecond term; and the diſcord, or minor ſeventh, deſcends a ſemitone major into the third, to root 3, the ſecond term.

From the motion of theſe two ſemitones, we may obſerve two things of equal importance:

1. That all original diſcords muſt be conſidered as minor ſounds, and muſt deſcend to the next degree to be reſolved.

2. That as the major third aſcended, ſo in like manner all major ſounds muſt aſcend to the next degree, and cannot deſcend without changing the ſcale by introducing a new term.

From this natural motion in the minor and major founds, we are conducted with the greateft certainty to the fecond term, or principal generator; as if by its powerful attraction it drew, and refolved every found to itfelf, in order to finifh the phrafe with the moft perfect termination.

From the vaft variety found in harmonics, we may have as many baffes as there are notes or letters in the combination. Thus this minor feventh, by changing the pofition from G, B, D, F, to B, D, F, G, or D, F, G, B, or to F, G, B, and D, forms other harmonics; but the firft only is fundamental; and the other difcords, which proceed from thefe inverfions, are formed by the different pofitions of the relative baffes, and diftinguifhed by particular figures placed over each, to fhew their relation to the fundamental bafs.

Thefe different baffes diverfify the proportions; and by placing the fame thing in a variety of forms, the firft impreffion is changed to others no lefs characteriftic and entertaining. Thus G, the fundamental, may rife a major third to B with a fixth and imperfect fifth, $\frac{6}{5}\flat$; or to D, a fifth above, with a compound fixth, 6, or, to F, a feventh above with a major fourth, or tritonus, and a fecond, $\frac{4+}{2}$.

Thefe tranfpofitions of the original combination do not alter the nature of the difcord at F, or the major found at B, fince in every pofition the F muft defcend to the next degree to be refolved, and the B muft afcend a femitone at the inftant

inftant the bafs moves from 9, the third term, to 3, the fecond.

Hence we difcover the difference between an original difcord and others which are accidental; for if we take away the minor feventh F, we remove at the fame time all the accidental difcords, viz. the femidiapente or imperfect fifth, the fourth or tritonus, the fecond, and the deficient third, 32, 27; and the fimple harmonics only remain, as at firft, in their moft perfect form.

I muft obferve that as melody depends fo much on a diatonic fucceffion of founds, which have been difcovered to be the immediate products of the fundamental baffes, it appears here, that by the affiftance of this minor feventh, we have a fyftem formed by a communication from the octave of the third term to its feventh or minor found, which proceeds to the third of the fecond term, as thus, G, F, E.

This fucceffion is not only very agreeable in itfelf, but gives a character to the feventh, which cannot be allowed to either of the fufpenfions, the ninth or the fourth; that is, this feventh may be ufed without being prepared, though, like the other two difcords, it muft be refolved by defcending to the next degree.

I have thus explained the moft remarkable parts of thefe combinations, by which the character of an original difcord is diftinguifhed from thofe that are accidental; together with the active principles of the two ſemitones, which direct the motions of the minor and major ſounds, ſo as to determine the motions of the original, as well as the accidental, diſcords in their different poſitions; by which we are conducted to the ſecond term, or principal generator, and its harmonics, which are the ſeveral points to which theſe motions are directed, as the final or perfect repoſe.

Thus the ſame laws, which inſtruct us in the *formation* of the maſs of harmony, teach us alſo how to give it *life* and *motion*; and if we attend to the ſeveral impreſſions made on our ear, by this diſſonant combination, tranſpoſed to different poſitions, we diſcover that our expectation of the ſucceeding ſecond term almoſt anticipates the change, and conducts the ear to its effect.

Nothing can be more pleaſing to a curious and intelligent mind, than to diſcover the mutual agreement between the laws of ſcience, and thoſe dictated by our own feelings; to be able to account for thoſe charming and agreeable ideas raiſed in the mind by the powers of harmony, and to reconcile them by principles the moſt rational and natural.

I ſhall now examine the other ſeventh, E, G, B, D, in the ſixth column, Pl. XXVIII, in order to diſcover by its motion the term that ſucceeds to it.

The ratios of the ſeveral intervals, which form this combination, have already been mentioned, in comparing them with thoſe of the third term, and the only difference appears to ariſe from the two ſpecies of thirds next to the fundamental baſs.

The

The ſeventh, like the other, muſt deſcend to be reſolved: but the minor third has no motion like the former, but muſt continue a ligature at the time of the reſolution, when the fundamental baſs deſcends a fifth, by which another ſeventh is formed. This indeed changes the poſitiòn of the harmonics, but with reſpect to the fundamental baſs, they ſtand in the ſame relation as before, and will continue the ſame ſo long as the baſs deſcends a fifth, at the time of the reſolution, which moves into a minor third.

Now in a progreſſion like this, where nothing is concluſive, and where even the ſcale is not always known, ſome eſſential alteration in the harmonics to make them more deciſive, by bringing us nearer to the ſecond term, is abſolutely neceſſary; and as a major ſound is the only one diſcovered proper to produce this alteration, the laſt ſeventh muſt be reſolved into a ſemitone major; by which means we have a major third to the ſucceeding baſs a fifth lower, which gives us the third term.

Theſe harmonics of the third term bring us home to the ſecond term, by the next motion of the fundamental baſs a fifth lower.

Thus by the motion of this ſeventh, we not only diſcover the term that ſucceeds to it, but alſo its character, for as it is antecedent to the third term in the ſame manner as the third is to the ſecond, it muſt therefore be a fourth term.

From this additional term, we have two ſorts of compariſons in the ratio of 3.

The

The first is, as the first term is to the second, so is the second to the third, which terminates in an imperfect repose, with the simple harmonics on the third term.

The other is, as the fourth term is to the third, so is the second, which finishes and completes the final or perfect repose.

The progression of the fourth term may be continued in one scale, or may move into any other agreeably to the nature of the mutations; but must finally be resolved into the third term, which is decisive with respect to the scale, by being constantly succeeded by the second term.

It appears therefore, that the lengthening of the phrase is the consequence of the use of the fourth term, and in proportion to their number will be the length of the phrase: for as no progression can be formed by either of the three terms in the scale, it is absolutely necessary to find that power placed some where, not only for the sake of the phrase, but also to vary the sentiment, by combinations less perfect, and less conclusive, than the others: all of which are found in the fourth term only, by being farther removed from the second term.

As this fourth term, like the former, is a compounded harmonic, it has also four positions. Thus E, G, B, D, a seventh, or G, B, D, E, a sixth and fifth $\frac{6}{5}$, or B, D, E, G, a fourth and third $\frac{4}{3}$, or D, E, G, and B, a second and fourth $\frac{4}{2}$.

In

In all theſe poſitions D is the diſcord, and muſt deſcend to the next degree to be reſolved; and if it moves a tone, it forms another fourth term, but if it deſcends a ſemitone, it forms a third term on a baſs or root, a fifth lower.

As the ratios of all compound harmonics are formed from the products of the three firſt terms differently combined, ſo thoſe of the fourth term muſt vary, according to the part of the ſcale from whence they are taken; for as no other ſounds can be admitted into their compoſition, the formation of the ſcale is of the greateſt importance in a theory of harmonics.

This diſcord of the fourth term is ſometimes uſed in a deſcending ſyſtem, without being prepared, as was before obſerved of the diſcord on the third term; but like that it muſt be reſolved.

From the many obſervations made on the third and fourth terms, it does not appear that their diſtinct characters are occaſioned by the ſeventh, when conſidered ſimply as a diſcord, but from its being united with a major or minor third.

It is from this union that the ſame ſpecies of diſcord diſcovers a much greater and more intereſting variety of revolutions in harmonics than either of the ſuſpenſions; and as the ſeventh is ſo immediately concerned in all theſe operations, that even the fundamental baſs is limited to deſcend a fifth, and ſometimes a third, at the time of its reſolution, and as the motion of the harmonics are regulated by the ſame cauſe, it follows that the ſeventh

ſeventh, from its extraordinary powers, muſt be conſidered as the *firſt* ſpecies of diſcord.

From this firſt ſpecies we have diſcovered a new term, which, by the great influence of the ſecond term in every ſcale, is placed as antecedent to the third term.

This diſpoſition determines it to be a fourth term, dependent in its reſolution on the third; in the ſame manner as the third is on the ſecond term. It appears therefore that, by the different applications of this firſt ſpecies of diſcord, and by attending to the motion and character of the fundamental baſs, the harmonic powers are greatly encreaſed.

Hence we have the ſimple and compounded harmonics limited to the different baſſes to which they belong: and if the ſame faithful guide continues to govern and direct the future operations, the double compounded harmonics will be diſcovered, and finiſh the characters of the third and fourth terms: as appears in the ſeventh column of Pl. XXVIII, and XXIX.

In this theory the motions of the fundamental baſſes are limited to that of aſcending or deſcending a fifth, and ſometimes a third; but there is no inſtance of their moving a tone, or ſemitone: from whence the true character of the melody is preſerved in its natural motion of tones and ſemitones, whilſt that of the fundamental baſs demands a more maſculine modulation.

Therefore as long as no other harmonics are admitted, but ſuch as depend on the motion, and are the products, of the fundamental

damental basses, so long will these two opposite characters be preserved, namely, the systematic motion for melodies, and the diastematic for the fundamental basses.

SECTION VII.

Of the double compounded THIRD *and* FOURTH TERMS.

It has been explained before that the formation either of the simple or compounded harmonics is a composition of thirds, in an ascending direction from a given sound or fundamental bass. Thus as the simple harmonics are a combination of two species of thirds, so another third of either species being added to them forms the compound, or discord of the seventh; or, as before explained, the third or fourth term.

If therefore we advance one step further on the same principles, and add another third to the former, we shall discover the utmost limits of harmonic combination in the double compounded third or fourth term. This harmonic consists of a third of either species, according to the term to which it is applied; of a fifth, a seventh, and a ninth, either major or minor, agreeably to the scale, and the relation that the combination has to the second term.

As this combination, from the variety of its proportions, is one of the most dissonant, and at the same time the most proper for the darker shades of musical composition, it cannot be unworthy of a particular consideration; especially as no ſuch combination has yet appeared in any theory that I have ſeen, occaſioned by an opinion, that the perfection of the diapaſon is ſuch, as to contain all other intervals within itſelf, and that whatever exceeds it, is only a repetition, at the diſtance of an octave, of what exiſted before, by which all ſimple intervals are made compound by the addition of their octaves.

This reaſon, however ſpecious, when applied to a ſimple melody, or a ſingle ſucceſſion of ſounds, cannot be admitted with reſpect to combination, which depends on other principles, drawn from the character and motion of the fundamental baſs. But that I may not advance any thing new, in ſo eſſential a part of harmonics as this is, without ſupporting it by the moſt undeniable evidence, I ſhall for this purpoſe ſelect from this theory the following rules.

1. That the whole powers of harmonics are conducted by *four terms*.

2. That the three firſt are primary fundamental baſſes with perfect fifths, and generate their own harmonics, from which the ſcale is formed, and all other combinations peculiar to it.

3. That the fourth term has nothing original in its compoſition, but is formed from the products of the three firſt terms, and as its communication with the ſecond term is cut off by the third term, it therefore ſtands as a ſecondary fundamental baſs, of an inferior character to the third term, to which it muſt deſcend a fifth.

4. That

4. That the motion of theſe four terms in one ſcale is limited to the triple and quintuple proportion; but in no inſtance admits of any other, either in the ratio of 9, 8, tone major, or 16, 15, ſemitone major.

5. That the third and fourth terms may have compounded, or double compounded, harmonics; from which the firſt and ſecond terms are excluded.

6. That the firſt and third terms only have an immediate communication with the ſecond,

How far theſe rules will agree with the doctrine of keeping all harmonics in their *firſt* and *moſt ſimple poſitions* within the limits of the diapaſon, remains now to be examined.

The extreme flat ſeventh, and imperfect fifth, or ſemidiapente, with a minor third, form the combination B♮, D, F, A♭, (Pl. XXIX.) which now comes under conſideration, in order to diſcover its character as a fundamental baſs.

The only ſound in the whole ſcale, on which theſe harmonics can be formed, is the major ſound, which is a product of the third term, and an imperfect concord of a third major. Now if this major ſound can ſupport the character of a fundamental baſs, all harmonic combination will be confined to the limits of the diapaſon; but if on the contrary it is found to have no one requiſite proper for this character, the double compounded harmonic muſt be eſtabliſhed as the true and original combination.

It is determined by the firſt rule, that all harmonic combination is formed on four terms; therefore, if this major ſound could be ſuppoſed to be one of the firſt three primary baſſes, it muſt have a perfect fifth, according to the ſecond rule; which we find it has not: and if it is a fourth term, it cannot move to the ſecond term, agreeably to the third and ſixth rule; and if incapable of this motion as a fourth term, its two minor diſcords cannot be reſolved by any other motion. Hence we diſcover that a *major ſound* cannot be a *fundamental baſs*, becauſe its motion of a ſemitone major, 16, 15, is abſolutely contrary to the fourth rule, as well as the whole theory, which limits the motions of the fundamental baſs, to the triple and quintuple proportion only.

We may conclude from the fifth rule, that this combination is a part of a double compound; and as it has no other motion in the ſame ſcale, but into the ſecond term, it can be derived from no other than a third term double compounded, agreeably to the ſixth rule.

Thus we diſcover that theſe harmonics, excluding the fundamental baſs, exceed the limits of the diapaſon, by a ſemitone in a minor ſcale, and in a major ſcale by a tone.

From theſe obſervations the firſt idea of a diſcord is preſerved, which had its original form in the union of the firſt and third

third terms; to which is now added the harmonic third of the first term, not by any arbitrary will of our own, but by a necessity, which the motion of the fundamental bass obliges us to comply with.

The extreme flat seventh and imperfect fifth being found as parts of a double compound, peculiar to the third term, it may not be improper to give one example of their formation. In Pl. xxv, root 5, C 30, E ♮, G, are the simple harmonics of the third term, to which the first term B ♭, 270, and its minor third, D ♭, 225, are added, which are taken from the quintuple of root 9, the first term in the triple; and the whole forms a double compounded harmonic on the third term: when the bass moves to F 90, the second term, the two minor discords are resolved, and the major sound E rises a semitone, by which the harmonics of the second term are completed.

Also at Pl. xxvi, the quintuple of root 9, B 180, D ♯, F ♯, a third term, is by the addition of A, C, taken from the quintuple of root 1, the first term, formed into a double compounded third term. But as its motion to the second, at the quintuple of root 3, together with the resolution of its two discords, and the ascending of the major sound, are the same as above, they need not be repeated.

From the same example taken from each of the diagrams we discover how the combination of harmonic sounds and their several relations depend on the fundamental bass; that by its animating powers they receive life and motion; that the character of a third term double compounded is established, in preference

preference to a major ſound, which has no other motion than a ſemitone; which, for this and other reaſons before given, can never be a fundamental baſs.

There are two inſtances in which this combination deſerves our particular notice.

The firſt is with reſpect to the motion of the ſemitone major. The active principle of this interval was firſt diſcovered in the harmonics of 1 to 3, or from 3 to 9, and the contrary; in which one ſemitone was formed by the change of each of theſe harmonics; but when the third term was compoſed of its minor ſeventh, we had two ſemitones formed by its motion into the ſecond term; the one from a minor ſound deſcending, and the other from a major ſound aſcending, as has been obſerved before. But in the preſent caſe, of a double compounded third term into the ſecond, we have three ſemitones formed; the flat ninth and minor ſeventh deſcend a ſemitone as minor ſounds to be reſolved, and the major ſound aſcends another ſemitone into the ſecond term, or its octave, which is the ſame.

The ſecond inſtance relates to the ſyſtems, which make ſo great a part of the ancient harmonica.

Theſe ſyſtems are of different magnitudes; but the leaſt muſt conſtitute a minor or major third: which agrees with Euclid's definition, that "the leaſt ſyſtem contains more intervals than one," (p. 1.)

In moving therefore from the ſecond to the third term twice, with their ſimple harmonics, we have a ſyſtem of diateſſaron, or four

four founds in their next degrees in the melody, as E, D, C, B, beginning with the third of the fecond term, and ending with the third of the third term, fecond diagram.

Syftem	E,	D,	C,	B.
Roots	C,	G,	C,	G.

If the third term is compounded with its minor feventh, we have a fyftem of fix degrees, from the eighth of the third term to the third of the fame term defcending, while the third term moves to the fecond term at the refolution of the feventh, and repeats the fame motion, till the whole is finifhed.

Syftem	G,	F,	E,	D,	C,	B.
Roots	G,	G,	C,	G,	C,	G.

But when the third term is double compounded, we have a fyftem of feven degrees, while the third term repeats its motion to the fecond term, during which the fyftem defcends from the minor ninth of the third term to its third, which determines the largeft harmonic fyftem which can be formed on any two fundamental baffes.

Syftem	A♭,	G,	F,	E♭,	D,	C,	B.
Roots	G,	G,	G,	C,	G,	C,	G.

Thefe two laft remarks fhew us the gradual increafe of the harmonic powers, from the fimple to the double compounded third term, by the modulation of the femitones, and the variety and extent of the fyftems.

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The obſervations made in the minor ninth, added to the third term, will in all reſpects be the ſame if the ninth is major; except, that in the reſolution of the diſcords of the laſt two ſemitones only will be modulated, inſtead of three, which are found in the former; and that the firſt is formed from a ſcale with a minor third, the latter from one with a major; but their relation to the ſecond term is the ſame, becauſe each has a major third and perfect fifth to the fundamental baſs, without which, it can never ſtand as a third term.

Hence the relation that any combination has to the ſecond term muſt determine the character of the fundamental baſs; for if there is no immediate communication with the ſecond term, without firſt moving to the third, we can be in no doubt of its being a fourth term.

Thus if a double compounded harmonic has a minor third, it forms a double compounded fourth term, and as ſuch leads us by the natural motion of its harmonics to a third term, agreeably to what has been obſerved before; viz. that the impreſſion made on the mind by a fourth term, whether ſingly or doubly compounded, is ſo undetermined, that we wait with a kind of impatience, till relieved by the ſucceeding third term, which diſcovers at once the ſcale, the phraſe, the ſecond term, and whatever elſe is neceſſary for our information.

The particular compoſition of this double compounded harmonic, or fourth term, is formed on the ſame principles with the third term; with this difference only, that if the ſeventh is minor, the third muſt be minor, and not major; but if the ſeventh

venth is major, (which can never happen to the third term,) the third muſt alſo be major: as to the ninth, it may be a major, or a minor, third above the ſeventh, according to the ſcale.

The two diſcords, the ninth and ſeventh, are frequently uſed on the fourth, or third term, without being prepared, but nevertheleſs muſt be reſolved: by deſcending to their next degrees, the root or fundamental baſs in double compounded harmonics does not always appear, unleſs in compoſitions of four, five, or ſix parts; but its place is eaſily diſcovered by the motion of the harmonics.

SECTION VIII.

THE SECOND SPECIES OF DISCORD.

The principles of the third and fourth terms being eſtabliſhed, and ſupported by the motion of the fundamental baſs, the ninth, or *ſecond ſpecies* of *diſcord,* comes next to be diſcovered.

To underſtand its genius and character it will be proper to explain two laws, to which this diſcord, as a ſuſpenſion, is ſubject.

1. It muſt be prepared. 2. It muſt be reſolved.

By the preparing of a diſcord is to be underſtood, that the ſound exiſts as a concord one quantity of time, after which, by the motion of the fundamental baſs, the ſame ſound becomes a diſcord on the next quantity of time; and finally muſt be reſolved on the third quantity of time, by deſcending to the next degree, either a tone or a ſemitone.

Theſe operations depend on certain diſtinctions, to be obſerved in the diviſions of time.

There are various ſpecies of muſical time, but the diviſions of each are comprehended in two or three equal quantities in a bar, or their multiples.

Theſe bars are diſtinguiſhed by a diviſion of their equal parts into accented and unaccented. If a bar is compoſed of four equal quantities, the firſt and third are accented; the ſecond and fourth are unaccented. If there are three quantities in a bar, the firſt only is accented, and the other two are unaccented. The preparing of a diſcord therefore muſt be on the unaccented part.

The ſame ſound continues, or is a ligature, and forms the diſcord on the following accented part, after which it muſt deſcend to be reſolved on the ſucceeding unaccented part, either a tone or a ſemitone.

Hence it appears, that all diſcords, prepared and reſolved ,require three quantities of time; the firſt quantity prepares the ear for its exiſtence on the ſecond, and the third ſatisfies the ear by the ſweetneſs of its reſolution.

This diſcord, like the minor ſeventh, has its origin in the *union* of the firſt and third terms; as may be ſeen at Pl. XXVIII, in in the fifth column, where D is its fundamental baſs, with its perfect fifth and minor third; and again, at Pl. XXIX, in the fifth column, where F is the fundamental baſs, with its perfect fifth and major third.

As this diſcord now ſtands, we may obſerve that the three terms are employed in its formation, and that the ſecond term is placed a fifth from each of the extremes, to which a major or minor third to the fundamental baſs is added, which completes its harmonics.

The conſtruction of theſe two ninths is in all reſpects the ſame, except the minor third to the firſt, and the major to the ſecond; which not only makes this diſcord proper for the two ſcales, but alſo for the different parts of the ſcales.

By this we diſcover, that though its original is on a primary fundamental baſs, yet it may exiſt on a ſecondary fundamental baſs, and may therefore be applied not only to either of the three terms, but alſo to a fourth term, if properly prepared and reſolved.

But as the character and genius of every diſcord can only be diſcovered by its motion and relation to the ſecond term, it will be proper to compare this ninth with the minor ſeventh, to enable us to judge of its being a new ſpecies of diſcord, from its different preparations and reſolutions, as well as from its motion and relation to the ſecond term.

The ninth may be prepared by being a fifth, but not by being a third, to the fundamental baſs.

The ſeventh is prepared by being a fifth or third to the fundamental baſs.

The fundamental baſs continues two quantities of time to receive the reſolution of the ninth into the eighth.

The fundamental baſs falls a fifth, or a third, to receive the reſolution of the ſeventh into the third, or eighth.

The ninth may be applied to the firſt term, and after its reſolution the ſecond term may ſucceed.

The ſeventh cannot exiſt on a firſt term, without changing its character to a fourth term, and being obliged to move to other fourth terms, before it can arrive at the ſecond term.

The ninth may be applied to the ſecond term without changing its character, or the ſcale.

The ſeventh cannot be applied to a ſecond term without changing both.

The ninth muſt always be prepared, unleſs united with the ſeventh, as a double compounded harmonic.

The ſeventh is frequently uſed in deſcending ſyſtems without being prepared.

Theſe differences between the two diſcords muſt be owned to be very great, more eſpecially as the motion of the fundamental baſs, its character, and relation to the ſecond term, are ſo particularly determined.

This is the great object to which all harmonics ſhould be directed; and unleſs ſome diſtinction marks the phraſe, the ſentiment, and the diſtance of the fundamental baſs, which leads

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us to the principal generator, or ſecond term, our pleaſure muſt abate in proportion to the difficulties we have to encounter.

As all inverſions teach us to diverſify an original harmony, without deviating from the natural genius and character of the fundamental baſs, I would recommend a conſtant attention to the fundamental baſs, its motion and harmonics, particularly thoſe which precede any ſpecies of diſcord: the ſame attention muſt be paid to thoſe which ſucceed, and are the reſolution of the diſcords; for although theſe two ſpecies of diſcord, the ſeventh and the ninth, ſhould each be prepared by a fundamental baſs deſcending a fifth, yet, by an attention to the phraſe, to the ſounds that move, and thoſe which remain as ligatures, the ſpecies of diſcord will be eaſily determined. For if the third to the fundamental, which prepares the diſcord, is the ligature, we are ſure the ſeventh is the diſcord; but if the fifth of that fundamental baſs ſhould be the ligature, the ſecond ſpecies of diſcord, or ninth, muſt undoubtedly be applied.

Hence the diſcord is very early known by its preparation: as to its reſolution, which is of ſuch conſequence to the phraſe and the ſucceeding harmonics, we can be under no difficulty, if we remember that the natural motion of all original diſcords is to *deſcend* a tone or ſemitone, and obſerve the ſituation of the ninth, which by being placed one degree above the eighth of the fundamental baſs, muſt, by a kind of neceſſity, move to it; but the contrary of this is true with reſpect to the ſeventh, which is placed a note under the octave of the fundamental baſs, to which it cannot aſcend, but muſt wait for another fundamental baſs to be reſolved into a third.

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I muſt alſo obſerve, that although the ninth is ſometimes found on a ſecondary fundamental baſs, yet it has a more natural connection with the three primary, and for that reaſon has a near relation to the ſecond term. But it is not ſo with this ſeventh, which, as it conſtitutes the principal character of the fourth term, can never be otherwiſe applied, nor have any immediate relation to the ſecond term.

There is one obſervation more I muſt make with reſpect to the ninth, which is, that there is no inſtance of its being applied, where we may not with propriety apply the ſimple harmonics, if it were not to vary the ſentiment, by employing a double quantity of time. This enables us to ſuſpend the ninth, by which means the ſucceeding ſound arrives with a ſweeter and more graceful effect.

I hope therefore it will not be thought improper to diſtinguiſh this diſcord as the *ſecond ſpecies*, and to call it the ſuſpenſion of the ninth. From the many diſtinctions made by comparing it with the ſeventh, if properly attended to, it muſt be impoſſible to miſtake it, either with reſpect to its character, or the ſimilitude of the figuring, as applied to the relative baſſes; for its preparation and reſolution on fundamental baſſes are ſo peculiar to itſelf, as to prevent any poſſibility of its being taken for any other diſcord.

SECTION IX.

THE THIRD SPECIES OF DISCORD.

The Diateſſaron, or perfect fourth, intended now for the *third* ſpecies of diſcord, has before been demonſtrated as a perfect concord in the ratio of 3, 4.

How this duplicity of character can belong to the ſame interval is the ſubject of our preſent enquiry.

It muſt not be forgot, that the products of the firſt, ſecond, and third terms, fixed and determined the place of every ſound, which formed the ſcale, as well as all poſſible combination of conſonant ſounds; and that neither of theſe three terms alone could extend its powers to the formation of diſcord.

Their *Union* therefore in part, or in the whole, became abſolutely neceſſary for the attainment of this great end; and at the ſame time demonſtrates the perfection of a theory, which obliges us to draw every thing from its *original principles*.

This has in part been already explained in the two firſt ſpecies of diſcords, where the firſt term was added as a compound harmonic to the third term, and formed the minor ſeventh, or *firſt* ſpecies of diſcord; and by applying the third term, as harmonic to the firſt, the ſuſpenſion of the ninth, or *ſecond* ſpecies of diſcord was formed.

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From the ſame original principles this *third* ſpecies of diſcord has its exiſtence; for by the union of the firſt and third terms we have the ſuſpenſion of the fourth; ſee Pl. XXVIII and XXIX; which, by being unitedwith the fifth to the fundamental baſs, and ſubject to the laws before preſcribed to the ſuſpenſion of the ninth, muſt therefore be prepared and reſolved.

The formation of this diſcord depends on the motion of the fundamental baſs, with its ſimple harmonics riſing a fifth on an accented part of the bar. This motion not only prepares the diſcord, but determines its combination to be an eighth, fifth, and fourth.

The fourth is prepared by the eighth of the preceding fundamental baſs, and is ſubſtituted as a diſcord in the preſent combination, in lieu of the third, on an accented part of a bar, for one quantity of time. This ſuſpenſion of the fourth muſt deſcend to the next degree, on the next unaccented part of the bar, to reſtore the third to its natural place, and complete the ſimple harmonics,

Thus a ſuſpenſion appears to be only a power (ſubject to certain conditions) of preventing, or keeping back, the motion of one ſound for a time; at the expiration of which it is put into its natural ſtate, and then the harmonics are perfected.

This power, as applied to harmonics, contributes greatly to vary the ſentiment, by imploying three quantities of time with

with the ſuſpenſion, when two only would have been ſufficient without it.

This conſideration alone makes diſcords neceſſary, not only to diſtinguiſh the motion of the fundamental baſs, but alſo to mark the time, and enforce the expreſſion of the harmonics,

This ſuſpenſion, or third ſpecies of diſcord, is formed in each diagram; (ſee Pl. XXVIII and XXIX, column 4.) and may therefore be reſolved into a minor or major third, by deſcending a tone, or a ſemitone.

Theſe two different reſolutions will enable us to apply this diſcord to a fourth term, as well as to a third and ſecond; but, from the nature of the ſcale, it can never be applied to a firſt term; for as that is the only fundamental baſs below the ſecond term, and has no perfect fifth below, it cannot, conſiſtently with the ſcale and its character of a *firſt* term, admit any other diſcord but the ſecond ſpecies, or ninth.

The ſecond term is alſo limited to the ſecond and third ſpecies of diſcords only; viz. the ninth and fourth. But the third and fourth terms may receive either of the three ſpecies of diſcords; and the firſt ſpecies may be either ſingly or doubly compounded, agreeably to the diſtinct character of the third and fourth terms before deſcribed.

As theſe limitations of the diſcords diſtinguiſh the different terms to which they are applied; ſo, by a compariſon of their

 preparations

preparations and resolutions, their several characters will be sufficiently known to determine the relation of any fundamental bass to the second term, or principal sound.

The 7^{th}	is prepared by a	3^{d}, and the fundamental bass descends 5.
The 9^{th}		5^{th}, and the fundamental bass descends 5.
The 4^{th}		8^{th}, and the fundamental bass ascends 5.

The 7^{th}	is resolved into a	3^{d}, and the fundamental bass descends 5.
The 9^{th}		8^{th}, and the fundamental bass continues.
The 4^{th}		3^{d}, and the fundamental bass continues.

It has already been found, p. 178, that a fundamental bass preceding the third term a fifth higher, and descending a fifth to it, must be a secondary fundamental bass, or fourth term; so, if several fundamental basses precede the third term in this order of fifths, they must all be deemed so many fourth terms; for as they have nothing in their composition the least conclusive, either in relation to the phrase, or principal generator, till the third term appears, and discovers their remote relation to the second term, it follows, that the *phrase must necessarily* be lengthened, according to the number of fourth terms preceding the third and second term, which determines the phrase and the scale.

This third species of discord having been explained, together with its preparation by an eighth, and the motion of the fundamental bass ascending a fifth, I shall next describe it when prepared by a seventh. This preparation may be made by the seventh of a third or fourth term, by the fundamental bass bafs defcending a fifth, and may be refolved into a third, either major or minor, the bafs continuing on; or may be refolved into another feventh if the root defcends a fifth, and will have the beft effect if on a third term, in order to proceed to the fecond term, on which it may exift as a fourth; or it may be united with the whole harmonics of the third term, which will make a ninth a major feventh and fourth, and be refolved into the fimple harmony of the fecond term on the next quantity of time.

The different pofitions of the harmonics of this third fpecies of difcord are many, but with refpect to the figuring there can be only three; viz. $\frac{5}{4}$, $\frac{5}{2}$, $\frac{7}{4}$; and, as was before obferved, by a proper attention to the motion of the fundamental bafs that precedes this difcord, and difcovers its preparation by the fundamental bafs rifing five, there will be found fo great a difference from a difcord prepared by one falling five, as muft remove every appearance of difficulty.

From what has been above explained concerning the three fpecies of difcord, we find that they have received their original exiftence from the *union* of the *firft* and *third* terms; that by the motion of the fundamental bafs, which determines their preparations and refolutions, we difcover their true character, in what particulars they differ from each other, and their relation to the fecond term; and that they are original, becaufe they comprehend all other diffonances whatever by their inverfions, or the different pofitions of the harmonic baffes, without altering the rules before prefcribed for the motion of the fundamental bafs.

Before I draw to a conclusion, I would take notice of one very great mistake made by modern theorists, which, if they had determined the fact by the ratio, could not have escaped them. We meet with it in Kircher's third book, page 145, and in Zarlino, page 363, in the third book of his Institutioni; the same has been adopted by all succeeding writers. What I mean is their description of the enharmonic gender.

They have made the first interval in the tetrachord of the enharmonic gender a semitone minor, as B, B×, C, and E; instead of the enharmonic diesis, which is B, C♭, C♮, E. The deception of marking B with a single cross, instead of a double one, is equally exceptionable. By this mistake they have placed the enharmonic diesis the second interval in the tetrachord, when it ought to have been the first. But to prove this beyond a doubt, the tetrachord in question is in the first diagram, in which there is no B ⨳. See the harmonic tables, Pl. XIV, where B 128 in the triple, when compared with C♭ in the quintuple progression, root 25, is the enharmonic diesis in its true ratio $\frac{128}{125}$; to which if a semitone minor is added, $\frac{25}{24}$, they will make a semitone major, and the residue of the tetrachord will be a ditone, or third major, $\frac{5}{4}$, an imperfect concord, not peculiar to the enharmonic, as the same interval is found in the diatonic gender, in which the third minor is the incomposite interval, or residue of the chromatic gender.

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The CONCLUSION.

HAVING confidered at large in the *Firſt* Part of this Work the Principles of the Grecian Doctrine of the Harmonica, as maintained by the *Ariſtoxenians*, and in this *Second* Part as ſupported by the *Pythagoreans*, in the Doctrine of the Ratio, it may be uſeful to make a few ſummary reflections on theſe two very extraordinary theories. — The former, maintained by Ariſtoxenus, Euclid, and Bacchius Senior, cannot be examined with too great attention, as it abounds with whatever is curious and intereſting to the ſubject. The order and diſpoſition of the ſeveral ſyſtems, particularly the tetrachord, from which the ſcales are formed, together with the number of flat and ſharp dieſes, by which the minor and major ſcales are completed; above all the ſeven ſpecies of the diapaſon, in which the tetrachords are comprehended, and regulated by the conjunction and disjunction; are among the many inſtances of the perfection of this theory.

The meſe, or tone of disjunction, is of the greateſt conſequence; for it not only prevents two eights, two fifths, or two major thirds in ſucceſſion, but ſtops us from moving into another ſcale, unleſs by the intervention of an additional baſs as a temperament, which marks and diſcovers the alteration neceſſary for a new ſcale. —The genders ſupply the imperfect conſonances; viz. the major third in the enharmonic, and the minor in the chromatic gender; each of which being taken from the diapaſon, there remain the two ſpecies of ſixths; in the ſame manner as the diapente, added to the diateſſaron, completes the magnitude of diapaſon. By the ſame method the diatonic moll forms the interval of a ſuperfluous ſecond, which, together

together with the extreme flat ſeventh, completes the diapapaſon.

The great advantage of diſcovering theſe intervals contributes to the perfecting of this theory; for notwithſtanding the ſeven ſpecies of diapaſon contained them all, yet their formation would have been attended with great difficulty, and their magnitude muſt ſometimes have been exceptionable; as from B to D in the firſt diagram, and from F to D in the ſecond, are each a minor third deficient by a comma: yet it muſt be obſerved, that although the Ariſtoxenians did not admit of the ratio in their theory, their diviſion of intervals was ſtrictly conformable to it. Thus the hemitone minor and major, which form a part of the chromatic gender, when added together are the ſum of the tone minor, and not the tone major. As to the ſpiſs, or ſmaller intervals, they are ſo neceſſary to the mutations, that nothing could be perfect without them; and though they may be taken out of the different ſpecies of diapaſon, yet the genders regulate and determine the magnitude of all the leſs intervals, with much greater eaſe and certainty.

If again we examine the rules laid down for the mutations, they are ſimple and eaſy to be underſtood; for they are comprehended in the perfect conſonant mutation, to aſcend or deſcend a fifth; the imperfect mutation, to aſcend or deſcend a third; and the diſſonant mutation, to aſcend or deſcend a tone. Theſe mutations are regulated by adding one dieſis, or by taking one away from the firſt propoſed number; by which means the mode is formed and ſupplied with all its varieties.

In

In the First part of this Work I did not treat of the Difcords; not that I was without materials; but as a Second Part, fupported by the ratio, has fucceeded to the former, in that I have explained the three fpecies of original difcords, agreeably to the numbers applied by Pythagoras to difcover the true ratio of concord and difcord; viz. 6, 8, 9, 12. I was defirous of doing honour to his memory, as the firft we know of, who difcovered the magnitude of mufical intervals by the ratio; which very properly fucceeds the two diagrams of Gaudentius, having the fame principles applied to each.

But here I muft beg leave to paufe one moment, to pay a tribute of gratitude moft juftly due to the merit of that great and excellent theorift and profeffor of Mufic, the learned Dr. *Pepufch*; from whom, in an early part of life, I received the firft principles of the application of numbers to mufical intervals. As I loved and honoured him while living, fo I greatly lamented his death, which happened at a time when I ftood moft in need of his affiftance. His church mufic, preferved in the Ancient Academy, is his beft eulogy as a Compofer.

While I pay this tribute due to the memory of a departed friend, I am happy to embrace the opportunity of acknowledging the very friendly affiftance I have received in this work from the *Rev. J. Trebeck*, and the *Rev. G. S. Townley*.

I fhall

I ſhall now make ſome obſervations on the *Second* Part, or the doctrine of the Ratio, maintained by Guadentius, and by Pythagoras.

The great advantage of diſcovering the magnitude of all muſical intervals by the Ratio cannot but be agreeable to men of ſcience and lovers of truth. The Ratio has brought out of obſcurity the two diagrams of Gaudentius and removed ſeveral objections made by writers of the firſt reputation.

Theſe diagrams conſiſting of tones major and limmas occaſioned the firſt and principal objection to the Grecian harmonica, as ſeeming deſtitute of the imperfect concords, without which, it is impoſſible to have harmony, or even melody; which laſt the Grecians are acknowledged to have had in the greateſt perfection. With reſpect to the imperfect concords, it is very extraordinary that no writer, whom I have ſeen, ever attempted to diſcover them in the genders: for as to the ſeven ſpecies of the diapaſon, from whence the genders are formed, as well as all other muſical intervals, it was not poſſible to extract them from thence, unleſs the poſition and harmonious conſtitution of thoſe diapaſons, together with their *Inverſion*, had been firſt diſcovered, and perfectly underſtood by the moderns.

With reſpect to the objection above-mentioned to the diagrams of Gaudentius, it was ſo true, that nothing could remove it but the application of numbers, or the harmonic principles: theſe were no ſooner applied, than the clouds of darkneſs diſappeared, and the *great*, the *perfect* and *immutable* ſyſtems of the diagrams, inſtead of being (as was imagined) a ſcale of ſounds

ſounds intended for the voice, are found to be roots, principal tones, or keys of ſeveral ſcales, formed into ſyſtems, in the ratio of tones major and limmas. However extraordinary theſe diagrams may appear, yet (as before obſerved) the ſame exiſt in our own theory; for if the principal tones of different ſcales be placed in the ſame order of degrees, they will be each to the next in the ratio of tone major or limma. Hence all the objections made to the Grecian harmonica on account of theſe Diagrams, muſt vaniſh, or muſt in an equal degree affect our own: which would be a taſk too difficult for any one to prove.

It is very extraordinary that authors of the firſt reputation ſhould condemn the Grecians for having no imperfect concords, when they are ſo eaſily diſcovered in the three genders, even according to their own definitions. But authors are ſometimes tempted to take upon truſt, whatever they meet with in the works of men of character, rather than give themſelves the trouble to diſcover the truth.

Five has been rejected as one of the primes, though the enharmonic dieſis (which they all agree is a part of that gender) cannot be formed without it. For as 125 is a multiple of 5, in the quintuple proportion, it muſt therefore be one of the primes. The quintuple proportion is ſometimes uſeful in the mutations; but as it is incapable of a progreſſion, it muſt always be ſucceeded by the triple proportion. Though in neither of theſe proportions can the mutations of the enharmonic dieſis, or of the limma, be found: the former is not applied in combination, nor mentioned by Euclid, when he treats of the mutations; which is a confirmation that it was only neceſſary to determine the exact magnitude of the ditone, or major third, in the enharmonic gender; as the chromatic was with respect to the minor third in the chromatic gender. The limma cannot be applied in combination, nor in the mutations, and therefore can be found only in the diagrams, being absolutely necessary for their formation, as systems of principal sounds.

I would here observe, that in the course of this work it has been shewn, that the fundamental bass cannot without a temperament ascend a tone or semitone, nor descend a tone or semitone; yet exceptions to these general laws are found in modern compositions in distinct phrases, in the Cadenza fugita, and in extra-harmonic sounds, which balk the cadence, and interrupt the regular progression of the fundamental bass.

These, and many other remarks which have been made in the course of this work, seemed to me necessary to remove every objection that could materially affect any essential part of the *Grecian Harmonica*: in which we discover the laws of science so intimately united and blended with those of nature, that the least deviation from the latter must in an equal degree be an injury to the simplicity of the former. That simplicity excludes the necessity of a profound knowledge in Geometry, in order to discover the original of the minor and major scales, of concords, discords, genders, modes, and other parts of the ancient Harmonica, which have long been perplexed and obscured by spheres, squares, cylinders, cones, and other geometrical figures, that have contributed little or nothing to the improvement of the practical Musician.

The *Ariſtoxenians* diſcovered the Agreement of concord and diſcord, and determined the magnitude of muſical intervals by the laws of their own natural Feelings.

The *Pythagoreans* applied themſelves to diſcover the *Ground* of thoſe laws; that by ſome leading and univerſal principle they might preſerve the voice of nature in its moſt pure and original ſtate. Hence they found that all melodies originated in, and were regulated by, the fundamental baſs. Hence their diagrams were confined to a geometric proportion in the ratio of 3: and the harmonious conſtruction of the tetrachord, the conjunction and disjunction of tetrachords, the minor and major ſcales, the flat and ſharp dieſes, were all regulated and determined by this firſt unerring principle, a progreſſion in the ratio of 3.

Here I cannot but take notice how generally this univerſal principle has regulated all muſical compoſitions, even to this day; and in many inſtances, where we cannot ſuppoſe it to be underſtood. Guido has placed his three hexachords (as was before obſerved) in the ſame ratio: and if we examine the three cliffs of the moderns F, C, and G, and the tuning of many ſtringed inſtruments, as in particular the Violin, Tenor, and Violoncello, their relation to each other is the ſame. Moreover the ſeveral ſpecies of diſcord (formed out of three terms by the union of the firſt and third terms in *combination*, whoſe motion could not be permitted in *ſucceſſion*) their preparations and reſolutions, the mutations with the ſame ſpecies of thirds, are all governed by the ſame ratio. Could all this be by Chance? rather, is there not an abſolute neceſſity, that All ſhall comply with principles founded in Nature? to whose laws we must be obedient, whether they are thoroughly understood or not.

Thus this wonderful theory of the Grecians, established on the most solid foundation, proceeds from a given sound, or principal generator, and advances with its creative powers, to generate its two products, the first and third terms, from whence arise all consonance and dissonance, scales and systems, major and minor, an encreasing series of flats and sharps, the tone major and minor, the different species of semitones, and all other intervals adapted to the mutations; the three genders, diatonic, chromatic, and enharmonic, the triple and quintuple progression of the fundamental bass, and whatever else is necessary to adorn, enrich and perfect the Science of Harmony.

It is almost impossible to enumerate all the advantages with respect to musical information and a profound knowledge of harmony, contained in the *Grecian Harmonica*. The simplicity and agreement of its several parts, established on laws that coincide with natural feelings and remain unalterable, are considerations sufficient to recommend it to all those who either wish to excel as Composers, or are desirous of attaining a *true knowledge* of the SCIENCE of MUSIC.

FINIS.

A

GENERAL INDEX

TO THE

THEORY

OF

HARMONICS.

GENERAL INDEX.

INTRODUCTION.

Alterations

FIRST PART.

I. Of SOUND.

II. Of INTERVALS.

Rational

III. Of the GENDERS.

IV. Of SYSTEMS.

V. Of the TONE, or MODE.

VI. Of

VI. Of the MUTATIONS.

Mutation

Mutation

Opinion

VII. Of the MELOPÆIA.

PART

PART THE SECOND.

INTRODUCTION.

Geometric

RULES relating to the RATIO.

PART II. SECTION I.

SECTION

SECTION II.

Of the TETRACHORD.

SECTION III.

Of the Conjunction and Disjunction* of TETRACHORDS.

SECTION IV.

Explanation of the QUINTUPLE PROPORTION.

Power

SECTION V.

Of the MUTATIONS.

SECTION VI.

Of DISCORDS.

Compounded

No

SECTION VII.

Of the double compounded *Third* and *Fourth* TERMS.

The

SECTION VIII.

The *Second* Species of DISCORD.

SECTION IX.

The *Third* Species of DISCORD.

Preparation

CONCLUSION.

References to thoſe Paſſages in which the *Plates* are explained.

XVII.

REFERENCES, &c.

ERRATA.

N.B. By a miſtake in *numbering* the Plates, No. XI. was omitted; in conſequence of which, Plate X. is immediately followed by Plate XII.

Page.	Line.		
24,	1,	for *of Harmony*,	read *of the Harmonica.*
27,	3,	for *dieſes*,	read *dieſis.*
31,	22,	for *dieſes*,	read *dieſis.*
50,	20,	for *Pate*,	read *Plate.*
72,	6,	for *different*,	read *ſufficient.*
100,	14,	for *next*,	read *leſs.*
117,	6,	for *progſion*,	read *progreſſion.*
131,	3,	for *Guadentius*,	read *Gaudentius.*
167,	10,	for *them otion*,	read *the motion.*
169,	7,	for Pl. VII.	read Pl. XXII.
173,	1,	for *the* ſeventh,	read *this* ſeventh.

Plate I.

Grecian Scale extracted from Euclid.

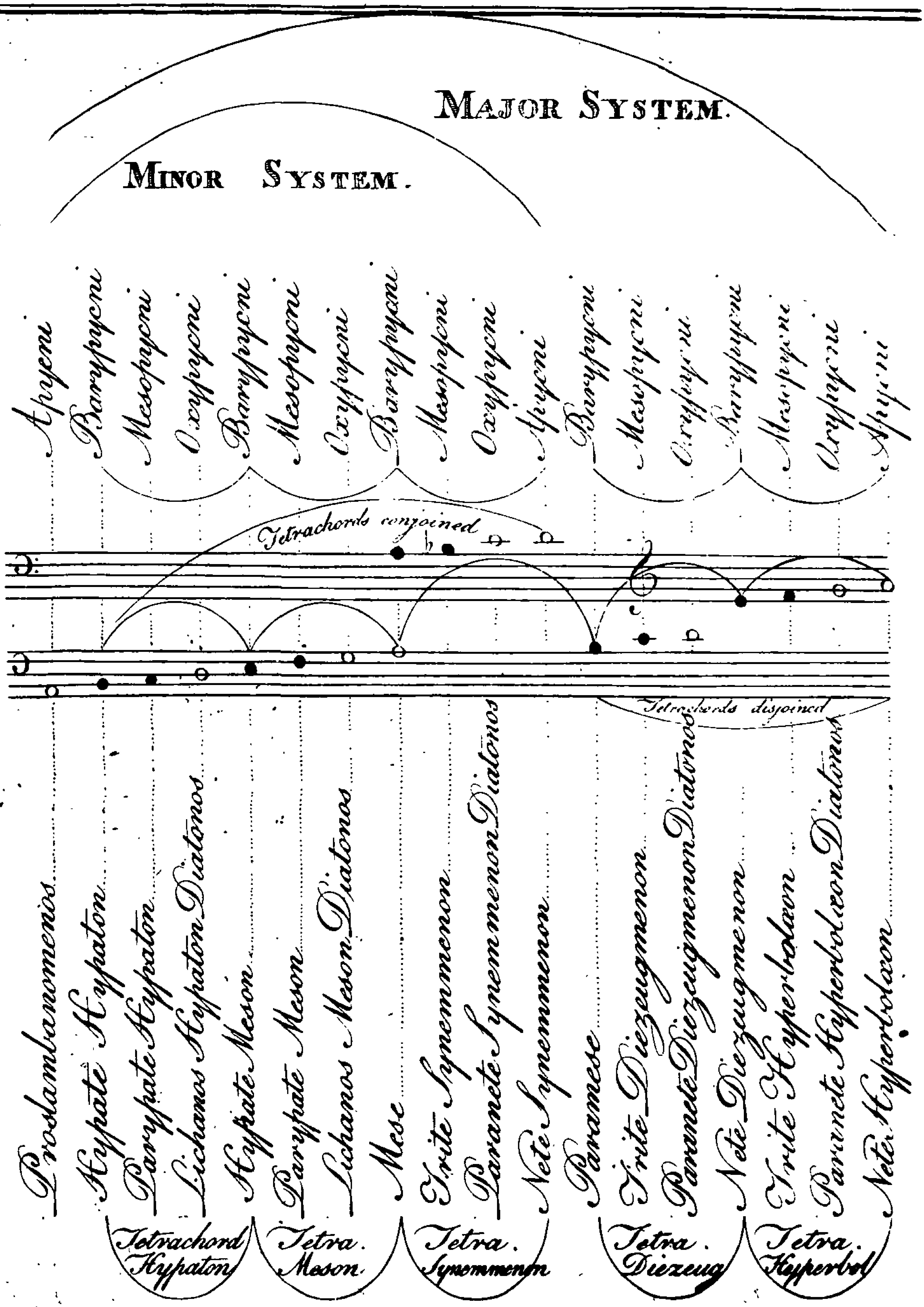
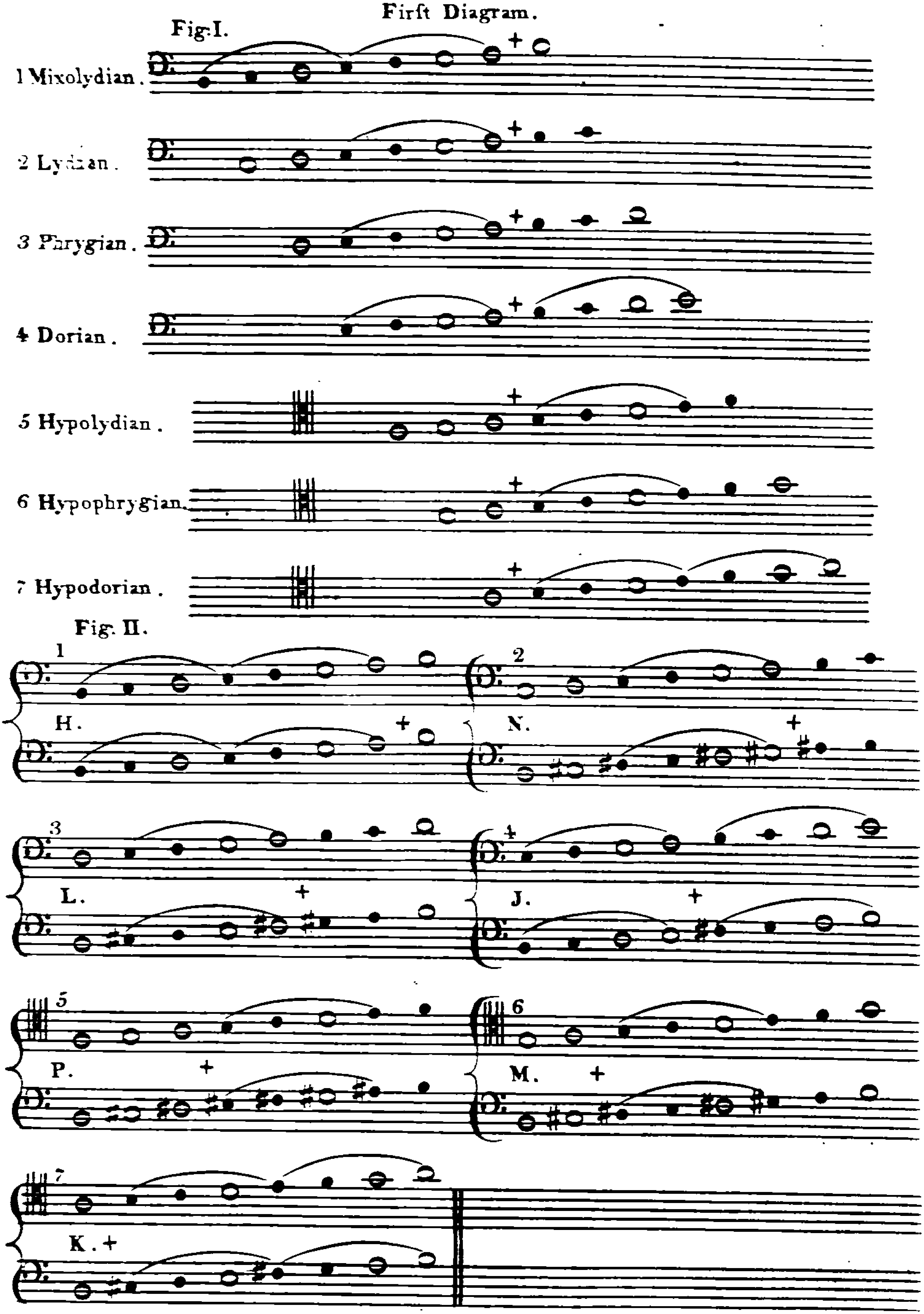


Plate II. **The seven Species of Diapason.**

First Diagram.



The Seven Species of Diapason.

Plate III

FIRST DIAGRAM.

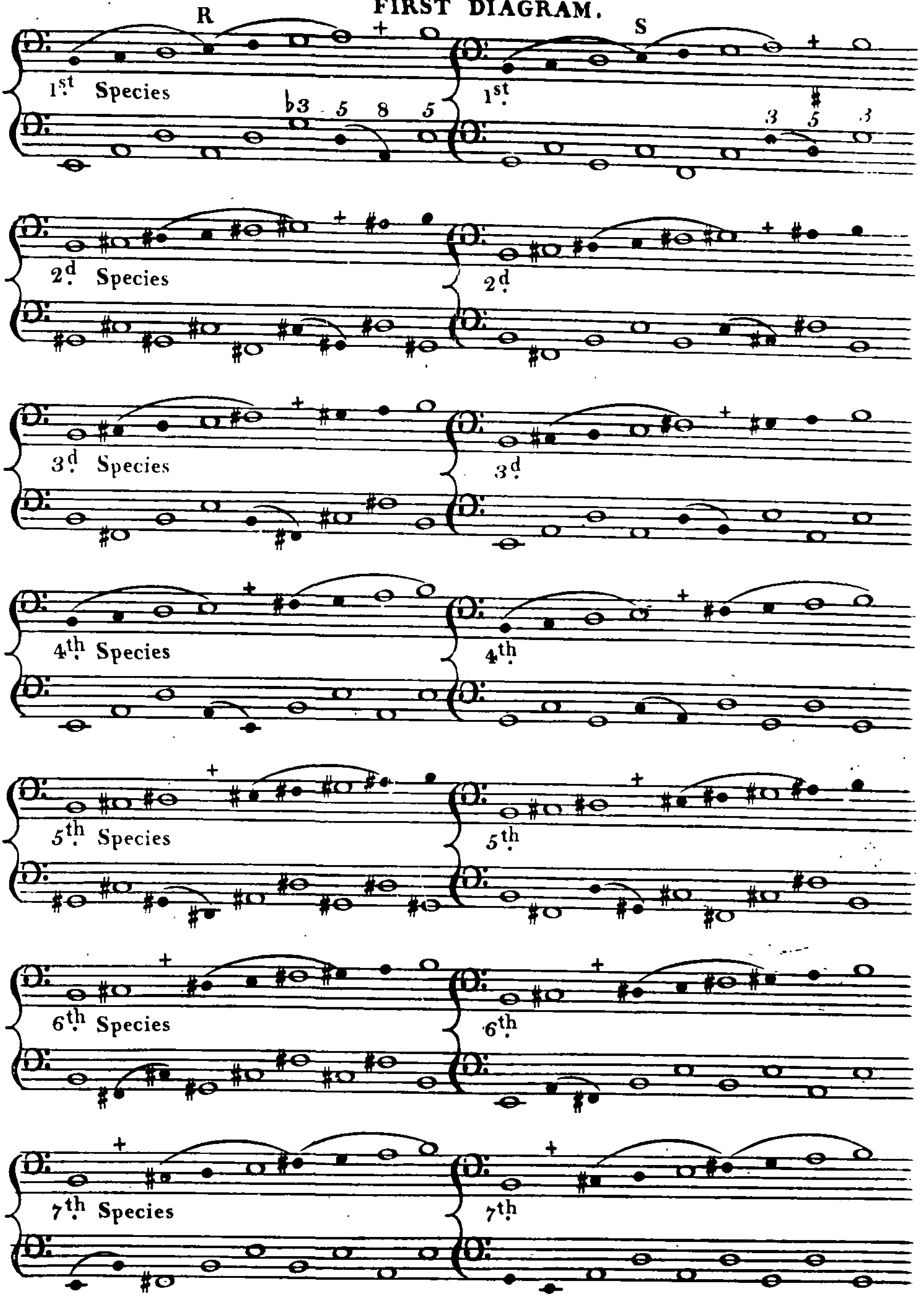


Plate IV.

EUCLID'S DIVISIONS of the TETRACHORD.
FIRST DIAGRAM.

EUCLID'S DIVISIONS of the TETRACHORD.

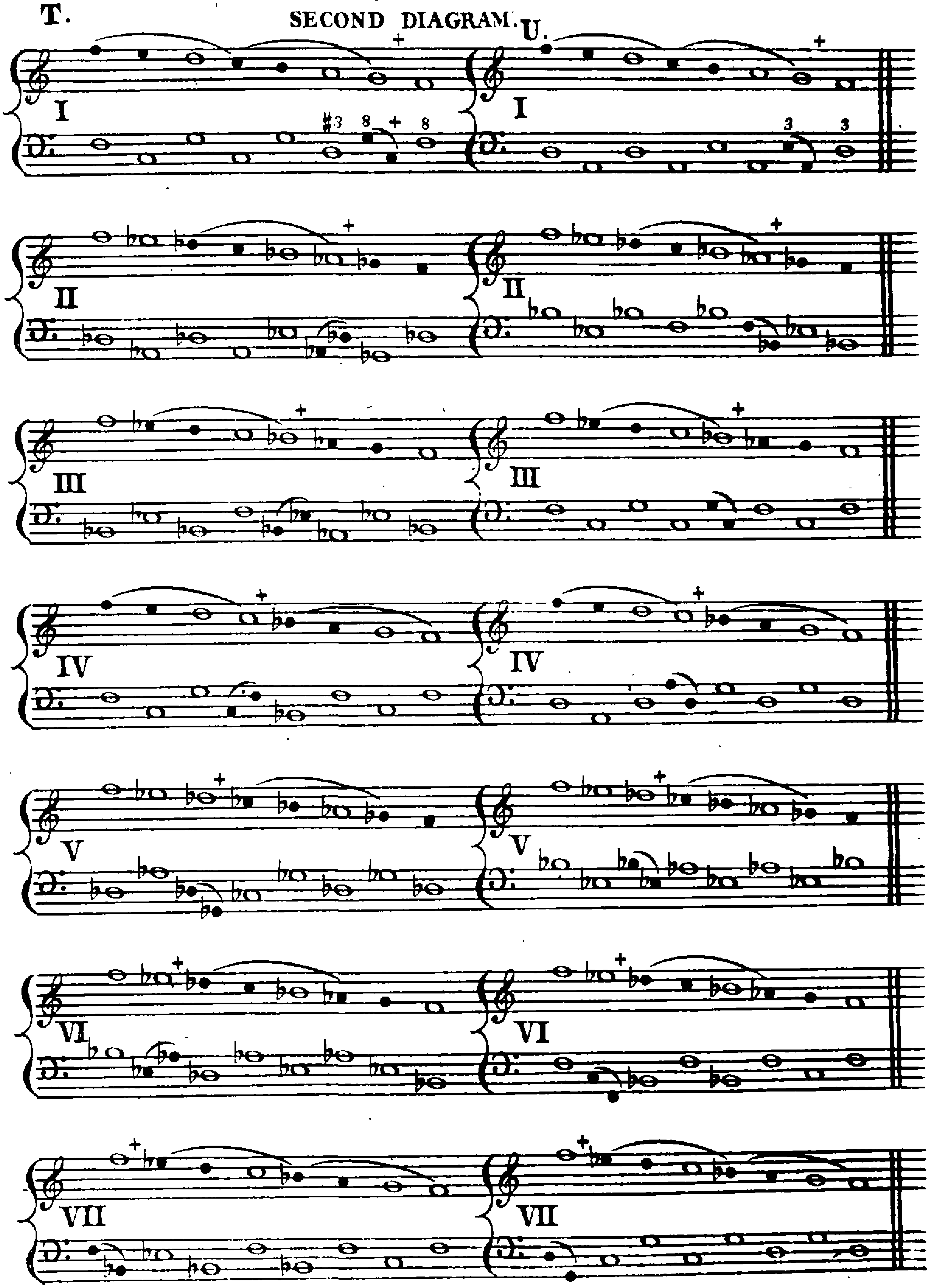
SECOND DIAGRAM.

Plate V.

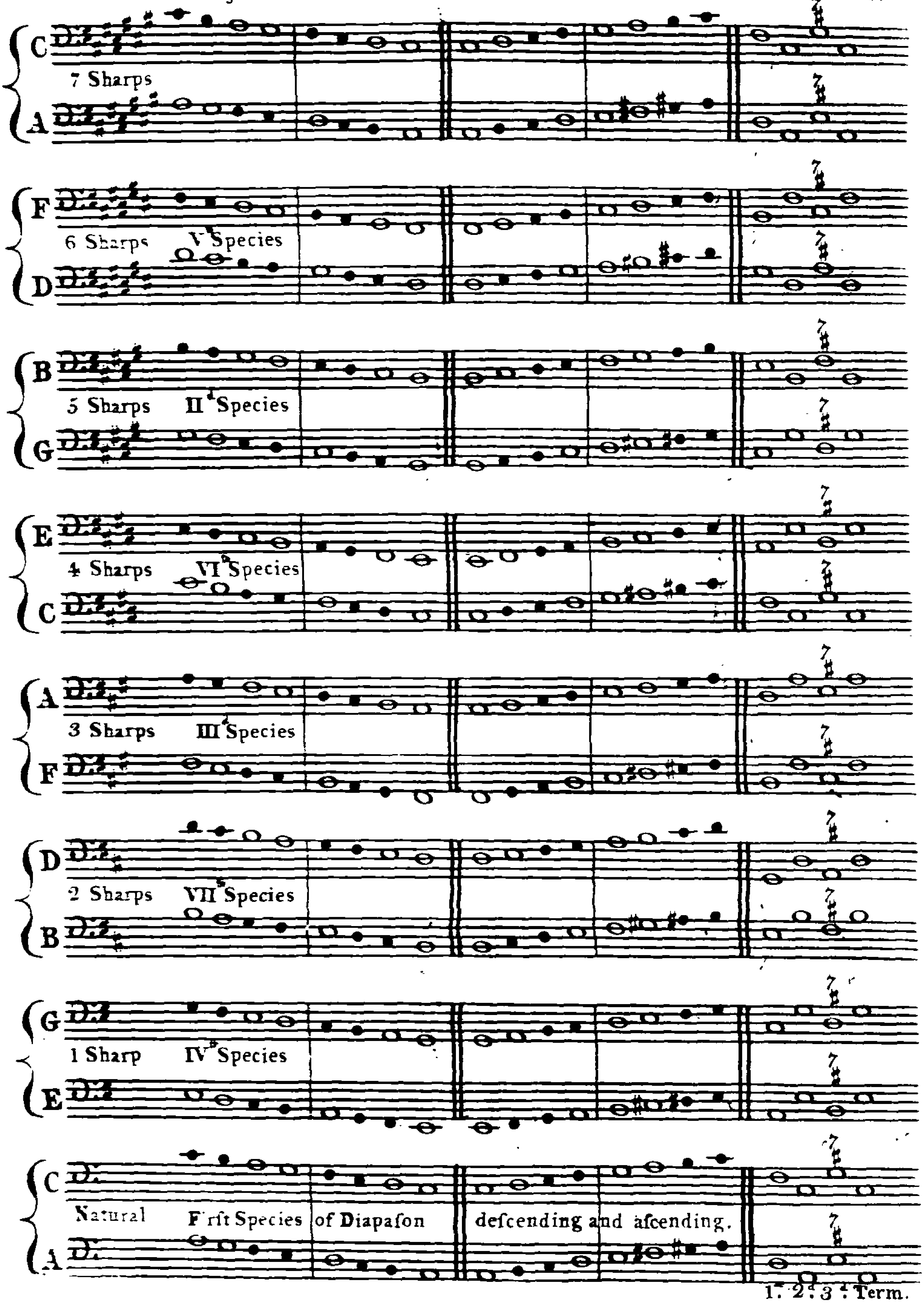
Plate VI. **Inversion of the Seven Species of Diapason.**

The Seven Species of Diapason.

SECOND DIAGRAM.



Scales with Major and Minor thirds extracted from the Harmonics of the Three Terms.



Scales. with Major and Minor thirds extracted from the Harmonics of the Three Terms.

Plate X.

Lengths of Strings descending from Unity, or from Acute to Grave.

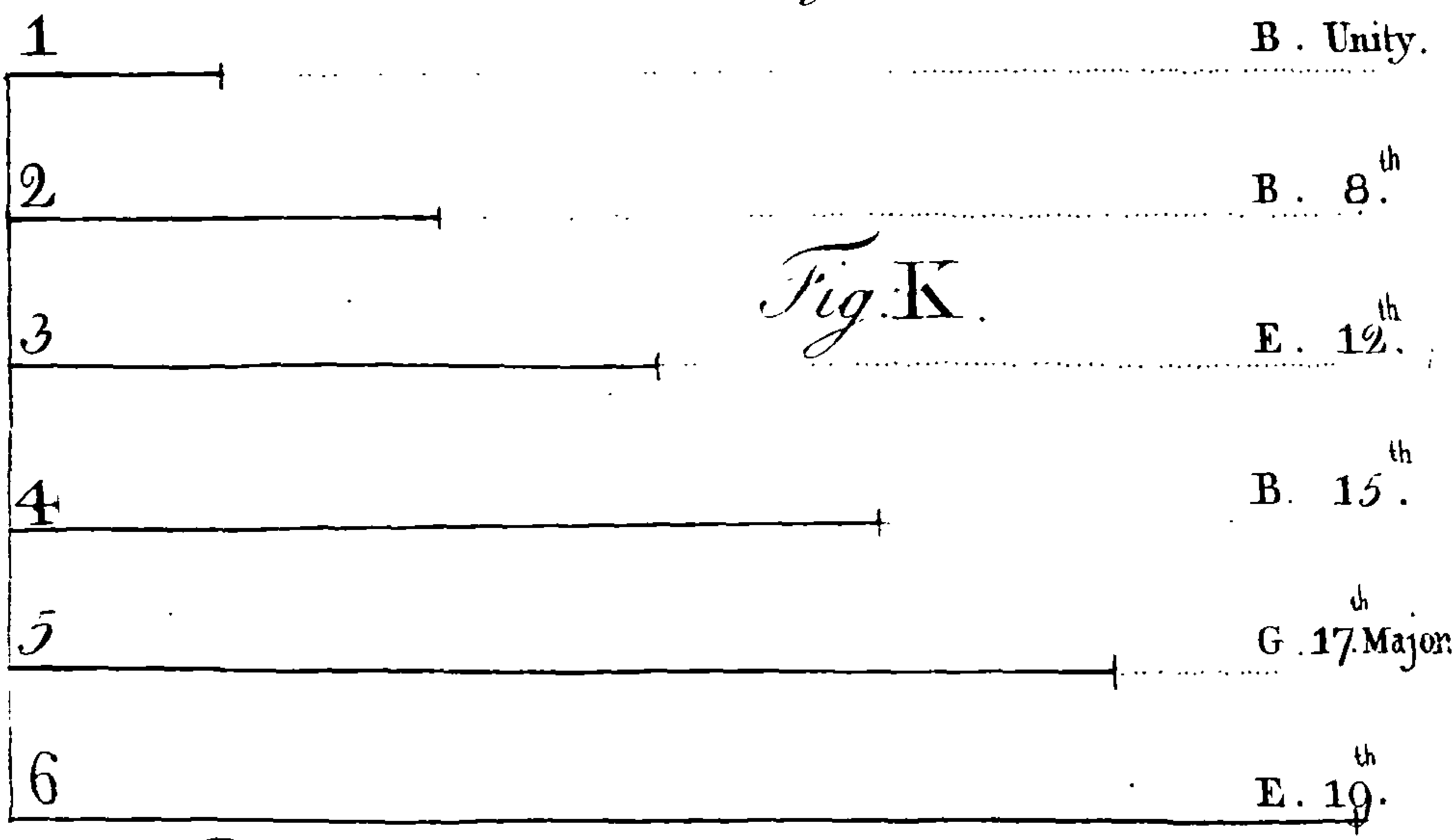


Fig. K.

Vibrations of Strings ascending from Unity, or from Grave to Acute.

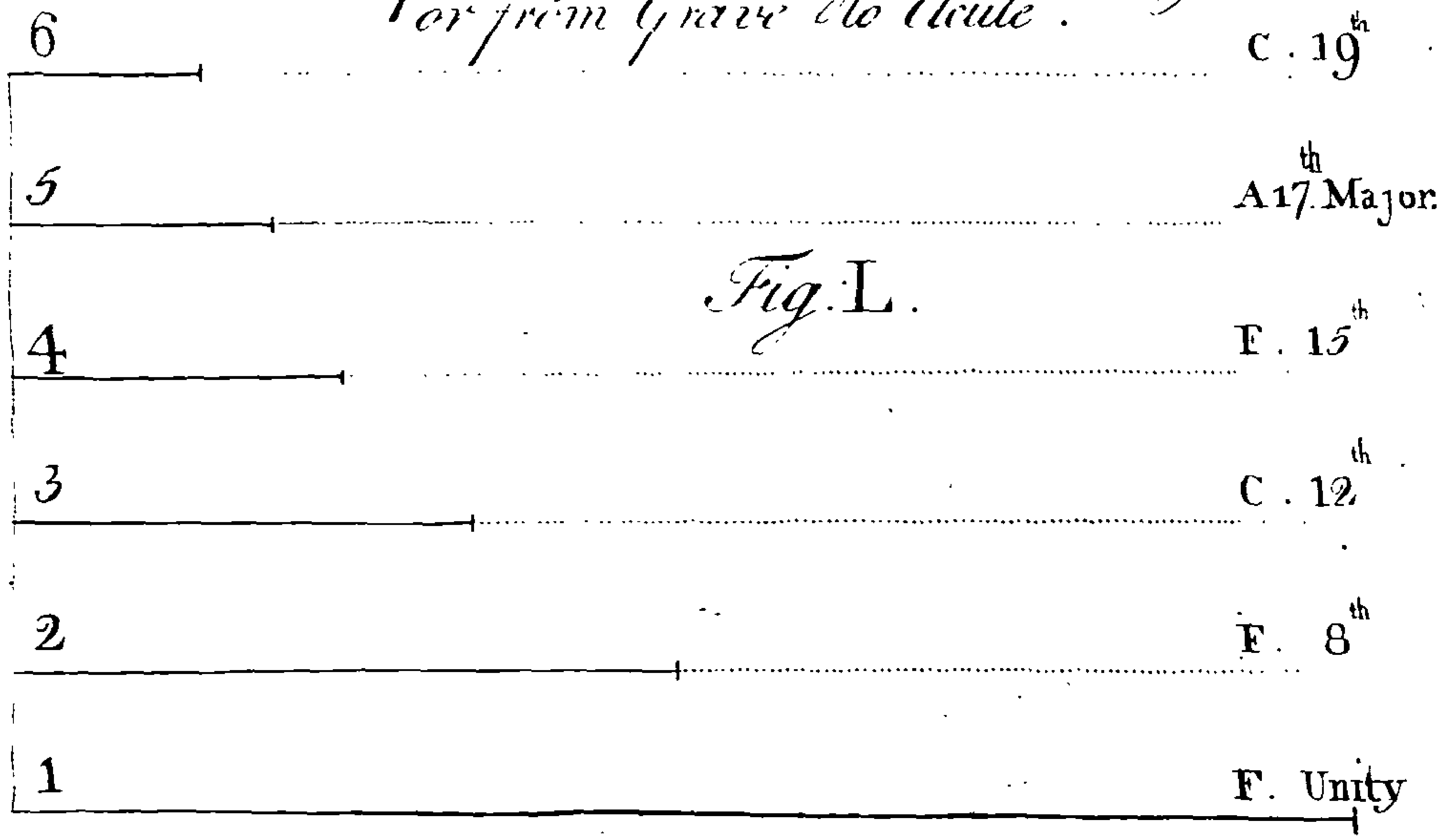


Fig. L.

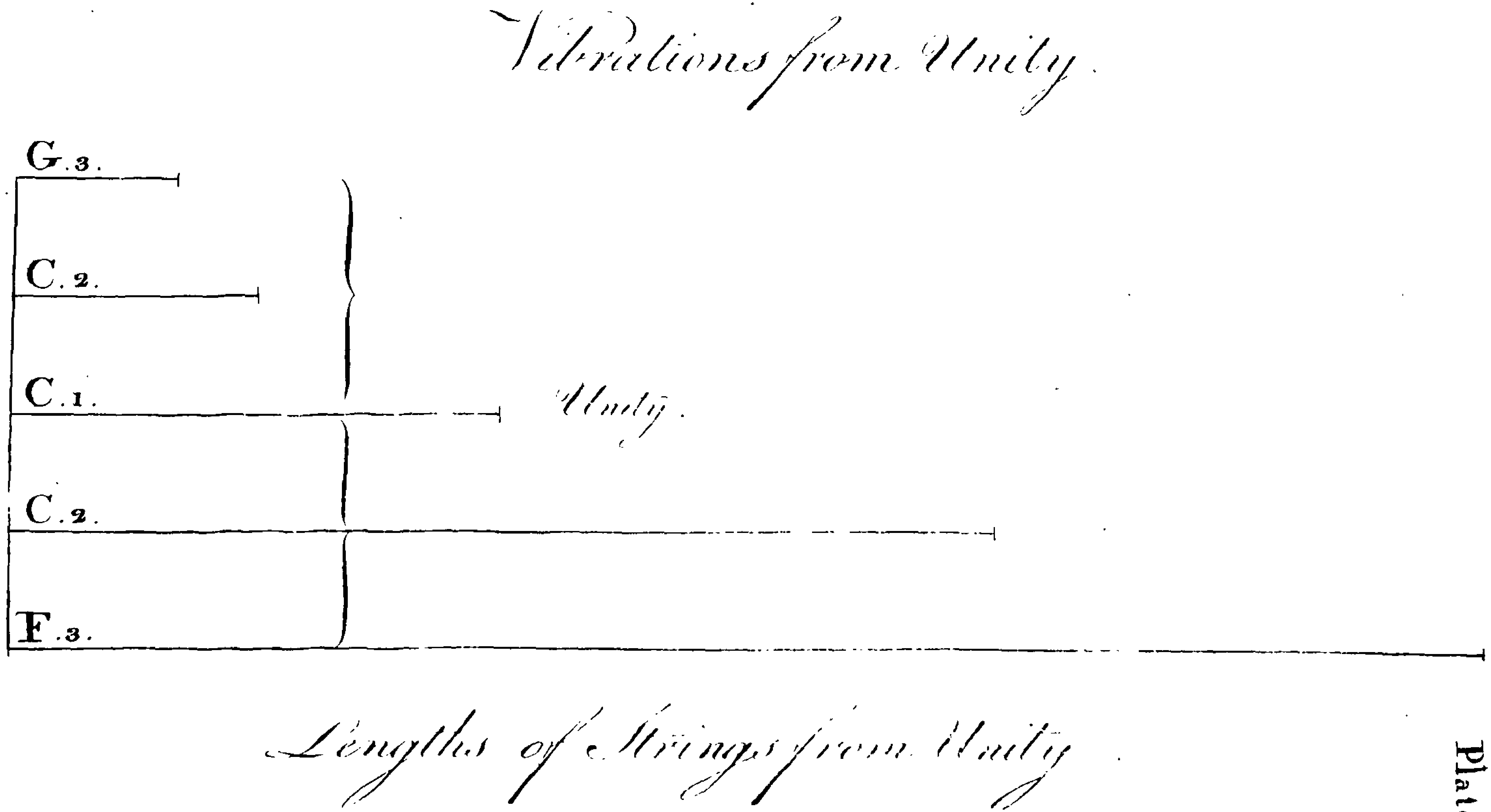

Vibrations from Unity.
G.3.
C.2.
C.1.
Unity.
C.2.
F.3.
Lengths of Strings from Unity.

Plate XII.

Plate XIII.

Harmonic Table.
Second Diagram.

Root 27 with its Harmonics.	8	D	216	432	864	1728	3456	6912	13824	27648	55296	110592
	6	A	162	324	648	1296	2592	5184	10368	20736	41472	82944
	5	F#	135	270	540	1080	2160	4320	8640	17280	34560	69120
	4	D	108	216	432	864	1728	3456	6912	13824	27648	55296
	3	A	81	162	324	648	1296	2592	5184	10368	20736	41472
	2	D	54	108	216	432	864	1728	3456	6912	13824	27648
	1	D	27	54	108	216	432	864	1728	3456	6912	13824
Root 9 with its Harmonics.	8	G	72	144	288	576	1152	2304	4608	9216	18432	36864
	6	D	54	108	216	432	864	1728	3456	6912	13824	27648
	5	B	45	90	180	360	720	1440	2880	5760	11520	23040
	4	G	36	72	144	288	576	1152	2304	4608	9216	18432
	3	D	27	54	108	216	432	864	1728	3456	6912	13824
	2	G	18	36	72	144	288	576	1152	2304	4608	9216
	1	G	9	18	36	72	144	288	576	1152	2304	4608
Root 3 with its Harmonics.	8	C	24	48	96	192	384	768	1536	3072	6144	12288
	6	G	18	36	72	144	288	576	1152	2304	4608	9216
	5	E	15	30	60	120	240	480	960	1920	3840	7680
	4	C	12	24	48	96	192	384	768	1536	3072	6144
	3	G	9	18	36	72	144	288	576	1152	2304	4608
	2	C	6	12	24	48	96	192	384	768	1536	3072
	1	C	3	6	12	24	48	96	192	384	768	1536
Root 1 with its Harmonics.	8	F	8	16	32	64	128	256	512	1024	2048	4096
	6	C	6	12	24	48	96	192	384	768	1536	3072
	5	A	5	10	20	40	80	160	320	640	1280	2560
	4	F	4	8	16	32	64	128	256	512	1024	2048
	3	C	3	6	12	24	48	96	192	384	768	1536
	2	F	2	4	8	16	32	64	128	256	512	1024
	1	F	1	2	4	8	16	32	64	128	256	512

continued

Second Diagram. Harmonic Table

continued

Root 2187ths Harmonics	8	F#	17496	34992	69984	139968	279936	559872	1119744
	6	C#	13122	26244	52488	104976	209952	419904	839808
	5	A#	10935	21870	43740	87480	174960	349920	699840
	4	F#	8748	17496	34992	69984	139968	279936	559872
	3	C#	6561	13122	26244	52488	104976	209952	419904
	2	F#	4374	8748	17496	34992	69984	139968	279936
	1	F#	2187	4374	8748	17496	34992	69984	139968
Root 729ths Harmonics.	8	B	5832	11664	23328	46656	93312	186624	373248
	6	F#	4374	8748	17496	34992	69984	139968	279936
	5	D#	3645	7290	14580	29160	58320	116640	233280
	4	B	2916	5832	11664	23328	46656	93312	186624
	3	F#	2187	4374	8748	17496	34992	69984	139968
	2	B	1458	2916	5832	11664	23328	46656	93312
	1	B	729	1458	2916	5832	11664	23328	46656
Root 243ths Harmonics.	8	E	1944	3888	7776	15552	31104	62208	124416
	6	B	1458	2916	5832	11664	23328	46656	93312
	5	G#	1215	2430	4860	9720	19440	38880	77760
	4	E	972	1944	3888	7776	15552	31104	62208
	3	B	729	1458	2916	5832	11664	23328	46656
	2	E	486	972	1944	3888	7776	15552	31104
	1	E	243	486	972	1944	3888	7776	15552
Root 81ths Harmonics.	8	A	648	1296	2592	5184	10368	20736	41472
	6	E	486	972	1944	3888	7776	15552	31104
	5	C#	405	810	1620	3240	6480	12960	25920
	4	A	324	648	1296	2592	5184	10368	20736
	3	E	243	486	972	1944	3888	7776	15552
	2	A	162	324	648	1296	2592	5184	10368
	1	A	81	162	324	648	1296	2592	5184

continued

Second Diagram Harmonic Table

continued.

Root 177147.	8	A♯	1417176
	6	E♯	1062882
	5	C♯♯	885535
	4	A♯	708588
	3	E♯	531441
	2	A♯	354294
	1	A♯	177147
Root 59049.	8	D♯	452392
	6	A♯	354294
	5	F♯♯	295245
	4	D♯	236196
	3	A♯	177147
	2	D♯	118098
	1	D♯	59049
Root 19683.	8	G♯	157464
	6	D♯	118098
	5	B♯	98415
	4	G♯	78732
	3	D♯	59049
	2	G♯	39366
	1	G♯	19683
Root 6561.	8	C♯	52488
	6	G♯	39366
	5	E♯	32805
	4	C♯	26244
	3	G♯	19683
	2	C♯	13122
	1	C♯	6561

Harmonic Table.
First Diagram.

Harmonics of Root 1.	1	B	1	2	4	8	16	32	64	128	256	512
	2	B	2	4	8	16	32	64	128	256	512	1024
	3	E	3	6	12	24	48	96	192	384	768	1536
	4	B	4	8	16	32	64	128	256	512	1024	2048
	5	G	5	10	20	40	80	160	320	640	1280	2560
	6	E	6	12	24	48	96	192	384	768	1536	3072
	8	B	8	16	32	64	128	256	512	1024	2048	4096
Root 3.	1	E	3	6	12	24	48	96	192	384	768	1536
	2	E	6	12	24	48	96	192	384	768	1536	3072
	3	A	9	18	36	72	144	288	576	1152	2304	4608
	4	E	12	24	48	96	192	384	768	1536	3072	6144
	5	C	15	30	60	120	240	480	960	1920	3840	7680
	6	A	18	36	72	144	288	576	1152	2304	4608	9216
	8	E	24	48	96	192	384	768	1536	3072	6144	12288
Root 9.	1	A	9	18	36	72	144	288	576	1152	2304	4608
	2	A	18	36	72	144	288	576	1152	2304	4608	9216
	3	D	27	54	108	216	432	864	1728	3456	6912	13824
	4	A	36	72	144	288	576	1152	2304	4608	9216	18432
	5	F	45	90	180	360	720	1440	2880	5760	11520	23040
	6	D	54	108	216	432	864	1728	3456	6912	13824	27648
	8	A	72	144	288	576	1152	2304	4608	9216	18432	36864
Root 27.	1	D	27	54	108	216	432	864	1728	3456	6912	13824
	2	D	54	108	216	432	864	1728	3456	6912	13824	27648
	3	G	81	162	324	648	1296	2592	5184	10368	20736	41472
	4	D	108	216	432	864	1728	3456	6912	13824	27648	55296
	5	Bb	135	270	540	1080	2160	4320	8640	17280	34560	69120
	6	G	162	324	648	1296	2592	5184	10368	20736	41472	82944
	8	D	216	432	864	1728	3456	6912	13824	27648	55296	110592

HARMONIC TABLE.

FIRST DIAGRAM.

continued

HARMONICS of ROOT 81	1	G	81	162	324	648	1296	2592	5184
	2	G	162	324	648	1296	2592	5184	10368
	3	C	243	486	972	1944	3888	7776	15552
	4	G	324	648	1296	2592	5184	10368	20736
	5	E♭	405	810	1620	3240	6480	12960	25920
	6	C	486	972	1944	3888	7776	15552	31104
	8	G	648	1296	2592	5184	10368	20736	41472
ROOT. 243.	1	C	243	486	972	1944	3888	7776	15552
	2	C	486	972	1944	3888	7776	15552	31104
	3	F	729	1458	2916	5832	11664	23328	46656
	4	C	972	1944	3888	7776	15552	31104	62208
	5	A♭	1215	2430	4860	9720	19440	38880	77760
	6	F	1458	2916	5832	11664	23328	46656	93312
	8	C	1944	3888	7776	15552	31104	62208	124416
ROOT. 729.	1	F	729	1458	2916	5832	11664	23328	46656
	2	F	1458	2916	5832	11664	23328	46656	93312
	3	B♭	2187	4374	8748	17496	34992	69984	139968
	4	F	2916	5832	11664	23328	46656	93312	186624
	5	D♭	3645	7290	14580	29160	58320	116640	233280
	6	B♭	4374	8748	17496	34992	69984	139968	279936
	8	F	5832	11664	23328	46656	93312	186624	373248
ROOT. 2187.	1	B♭	2187	4374	8748	17496	34992	69984	139968
	2	B♭	4374	8748	17496	34992	69984	139968	279936
	3	E♭	6561	13122	26244	52488	104976	209952	419904
	4	B♭	8748	17496	34992	69984	139968	279936	559872
	5	G♭	10935	21870	43740	87480	174960	349920	699860
	6	E♭	13122	26244	52488	104976	209952	419904	839808
	8	B♭	17496	34992	69984	139968	279936	559872	1119744

continued

First Diagram. Harmonic Table

continued

Harmonics of Root 6561.	1	E♭	6561
	2	E♭	13122
	3	A♭	19683
	4	E♭	26244
	5	C♭	32805
	6	A♭	39366
	8	E♭	52488
Root 19683.	1	A♭	19683
	2	A♭	39366
	3	D♭	59049
	4	A♭	78732
	5	F♭	78415
	6	D♭	118098
	8	A♭	157464
Root 59049.	1	D♭	59049
	2	D♭	118098
	3	G♭	177147
	4	D♭	236196
	5	B♭♭	295245
	6	G♭	354294
	8	D♭	472392
Root 177147.	1	G♭	177147
	2	G♭	354294
	3	C♭	531441
	4	G♭	708588
	5	E♭♭	885535
	6	C♭	1062882
	8	G♭	1417176

First Diagram.
Quintuple.

Root 5.	4	G	20	40	80	160	320	640	1280	2360
		E♮	24	48	96	192	384	768	1536	3072
	5	E♭	25	50	100	200	400	800	1600	3200
	6	C	30	60	120	240	480	960	1920	3840
Root 15.	4	C	60	120	240	480	960	1920	3840	7680
		A♮	72	144	288	576	1152	2304	4608	9216
	5	A♭	75	150	300	600	1200	2400	4800	9600
	6	F	90	180	360	720	1440	2880	5760	11520
Root 45.	4	F	180	360	720	1440	2880	5760	11520	23040
		D♮	216	432	864	1728	3456	6912	13824	27648
	5	D♭	225	450	900	1800	3600	7200	14400	28800
	6	B♭	270	540	1080	2160	4320	8640	17280	34560
Root 135.	4	B♭	540	1080	2160	4320	8640	17280	34560	69120
		G♮	648	1296	2592	5184	10368	20736	41472	82944
	5	G♭	675	1350	2700	5400	10800	21600	43200	86400
	6	E♭	810	1620	3240	6480	12960	25920	51840	103680
Root 405.	4	E♭	1620	3240	6480	12960	25920	51840	103680	207360
		C♮	1944	3888	7776	15552	31104	62208	124416	248832
	5	C♭	2025	4050	8100	16200	32400	64800	129600	259200
	6	A♭	2430	4860	9720	19440	38880	77760	155520	311040
Root 1215.	4	A♭	4860	9720	19440	38880	77760	155520	311040	622080
		F♮	5832	11664	23328	46656	93312	186624	373248	746496
	5	F♭	6075	12150	24300	48600	97200	194400	388800	777600
	6	D♭	14610	29220	58440	116880	233760	467520	935040	1870080

First Diagram.
Quintuple.

Root 25.	4	E♭	100	200	400	800	1600	3200	6400
		C♮	120	240	480	960	1920	3840	7680
	5	C♭	125	250	500	1000	2000	4000	8000
	6	A♭	150	300	600	1200	2400	4800	9600
Root 75.	4	A♭	300	600	1200	2400	4800	9600	19200
		F♮	360	720	1440	2880	5760	11520	23040
	5	F♭	375	750	1500	3000	6000	12000	24000
	6	D♭	450	900	1800	3600	7200	14400	28800
Root 225.	4	D♭	900	1800	3600	7200	14400	28800	57600
		B♭	1080	2160	4320	8640	17280	34560	69120
	5	B♭♭	1125	2250	4500	9000	18000	36000	72000
	6	G♭	1350	2700	5400	10800	21600	43200	86400
Root 675.	4	G♭	2700	5400	10800	21600	43200	86400	172800
		E♭	3240	6480	12960	25920	51840	103680	207360
	5	E♭♭	3375	6750	13500	27000	54000	108000	216000
	6	C♭	4050	8100	16200	32400	64800	129600	259200
Root 2025.	4	C♭	8100	16200	32400	64800	129600	259200	518400
		A♭	9720	18440	36880	73760	147520	295040	590080
	5	A♭♭	10125	20250	40500	81000	162000	324000	648000
	6	F♭	12150	24300	48600	97200	194400	388800	777600
Root 6075.	4	F♭	24300	48600	97200	194400	388800	777600	1555200
		D♭	29160	58320	116640	233280	466560	933120	1866240
	5	D♭♭	30375	60740	121480	242960	485920	971840	1943680
	6	B♭♭	36450	72900	145800	291600	583200	1166400	2332800

First Diagram.
Quintuple.

125.	4	Cb	500	1000	2000	4000	8000	16000
		Ab	600	1200	2400	4800	9600	19200
	5	Abb	625	1250	2500	5000	10000	20000
	6	Fb	750	1500	3000	6000	12000	24000
Root 37	4	Fb	1500	3000	6000	12000	24000	48000
		Db	1800	3600	7200	14400	28800	57600
	5	Dbb	1875	3750	7500	15000	30000	60000
	6	Bbb	2250	4500	9000	18000	36000	72000
Root 1125.	4	Bbb	4500	9000	18000	36000	72000	144000
		Gb	5400	10800	21600	43200	86400	172800
	5	Gbb	5625	11250	22500	45000	90000	180000
	6	Ebb	6750	13500	27000	54000	108000	216000
Root 3375.	4	Ebb	13500	27000	54000	108000	216000	432000
		Cb	16200	32400	64800	129600	259200	518400
	5	Cbb	16875	33750	67500	135000	270000	540000
	6	Abb	20250	40500	81000	162000	324000	648000
Root 10125.	4	Abb	40500	81090	162000	324000	648000	1296000
		Fb	48600	97200	194400	388800	777600	1555200
	5	Fbb	50625	101250	202500	405000	810000	1620000
	6	Dbb	60750	121500	243000	486000	972000	1944000

Plate XVI.

Second Diagram. Quintuple.

Root 1215.	6	D✱	7290	14580	29160	58320	116640	233280	466560	933120
	5	B✱	6075	12150	24300	48600	97200	194400	388800	777600
		B♮	5832	11664	23328	46656	93312	186624	373248	746496
	4	G✱	4860	9720	19440	38880	77760	155420	310840	621680
Root 405.	6	G✱	2430	4860	9720	19440	38880	77760	155520	311040
	5	E✱	2025	4050	8100	16200	32400	64800	129600	259200
		E♮	1944	3888	7776	15552	31104	62208	124416	248832
	4	C✱	1620	3240	6480	12960	25920	51840	103680	207360
Root 135.	6	C✱	810	1260	3240	6480	12960	25920	51840	103680
	5	A✱	675	1350	2700	5400	10800	21600	43200	86400
		A♮	648	1296	2592	5184	10368	20736	41472	82944
	4	F✱	540	1080	2160	4320	8640	17280	34560	69120
Root 45.	6	F✱	270	540	1080	2160	4320	8640	17280	34560
	5	D✱	225	450	900	1800	3600	7200	14400	28800
		D♮	216	432	864	1728	3456	6912	13824	27648
	4	B	180	360	720	1440	2880	5760	11520	23040
Root 15.	6	B	90	180	360	720	1440	2880	5760	11520
	5	G✱	75	150	300	600	1200	2400	4800	9600
		G♮	72	144	288	576	1152	2304	4608	9216
	4	E	60	120	240	480	960	1920	3840	7680
Root 5.	6	E	30	60	120	240	480	960	1920	3840
	5	C✱	25	50	100	200	400	800	1600	3200
		C♮	24	48	96	192	384	768	1536	3072
	4	A	20	40	80	160	320	640	1280	2560

Second Diagram.
Quintuple.

Root 6075.	6	F##	36450	72900	145800	291600	583200	1166400	2332800
	5	D##	30375	60750	121500	24300	486000	972000	1944000
		D#	29160	58320	116640	233280	466560	933120	1866240
	4	B#	24300	48600	97200	194400	388800	777600	1555200
Root 2025.	6	B#	12150	24300	48600	97200	194400	388800	777600
	5	G##	10125	20250	40500	81000	162000	324000	648000
		G#	9720	19440	38880	77760	155520	311040	622080
	4	E#	8100	16200	32400	64800	129600	259200	518400
Root 675.	6	E#	4050	8100	16200	32400	64800	129600	259200
	5	C##	3375	6750	13500	27000	54000	108000	216000
		C#	3240	6480	12960	25920	51840	103680	207360
	4	A#	2700	5400	10800	21600	43200	86400	172800
Root 225.	6	A#	1350	2700	5400	10800	21600	43200	86400
	5	F##	1125	2250	4500	9000	18000	36000	72000
		F#	1080	2160	4320	8640	17280	34560	69120
	4	D#	900	1800	3600	7200	14400	28800	57600
Root 75.	6	D#	450	900	1800	3600	7200	14400	28800
	5	B#	375	750	1500	3000	6000	12000	24000
		B♮	360	720	1440	2880	5760	11520	23040
	4	G#	300	600	1200	2400	4800	9600	19200
Root 25.	6	G#	150	300	600	1200	2400	4800	9600
	5	E#	125	250	500	1000	2000	4000	8000
		E♮	120	240	480	960	1920	3840	7680
	4	C#	100	200	400	800	1600	3200	6400

Second Diagram.
Quintuple.

Root 10125.	6	D##	60750	121500	243000	486000	972000	1944000
	5	B##	50625	101250	202500	405000	810000	1620000
		B#	48600	97200	194400	388800	777600	1555200
	4	G##	40500	81000	162000	324000	648000	1296000
Root 3375.	6	G##	20250	40500	81000	162000	324000	648000
	5	E##	16875	33750	67500	135000	270000	540000
		E#	16200	32400	64800	129600	259200	518400
	4	C##	13500	27000	54000	108000	216000	432000
Root 1125.	6	C##	6750	13500	27000	54000	10800	216000
	5	A##	5625	11250	22500	45000	90000	180000
		A#	5400	10800	21600	43200	86400	172800
	4	F##	4500	9000	18000	36000	72000	144000
Root 375.	6	F##	2250	4500	9000	18000	36000	72000
	5	D##	1875	3750	7500	15000	30000	60000
		D#	1800	3600	7200	14400	28800	57600
	4	B#	1500	3000	6000	12000	24000	48000
Root 125.	6	B#	750	1500	3000	6000	12000	24000
	5	G##	625	1250	2500	5000	10000	20000
		G#	600	1200	2400	4800	9600	19200
	4	E#	500	1000	2000	4000	8000	16000

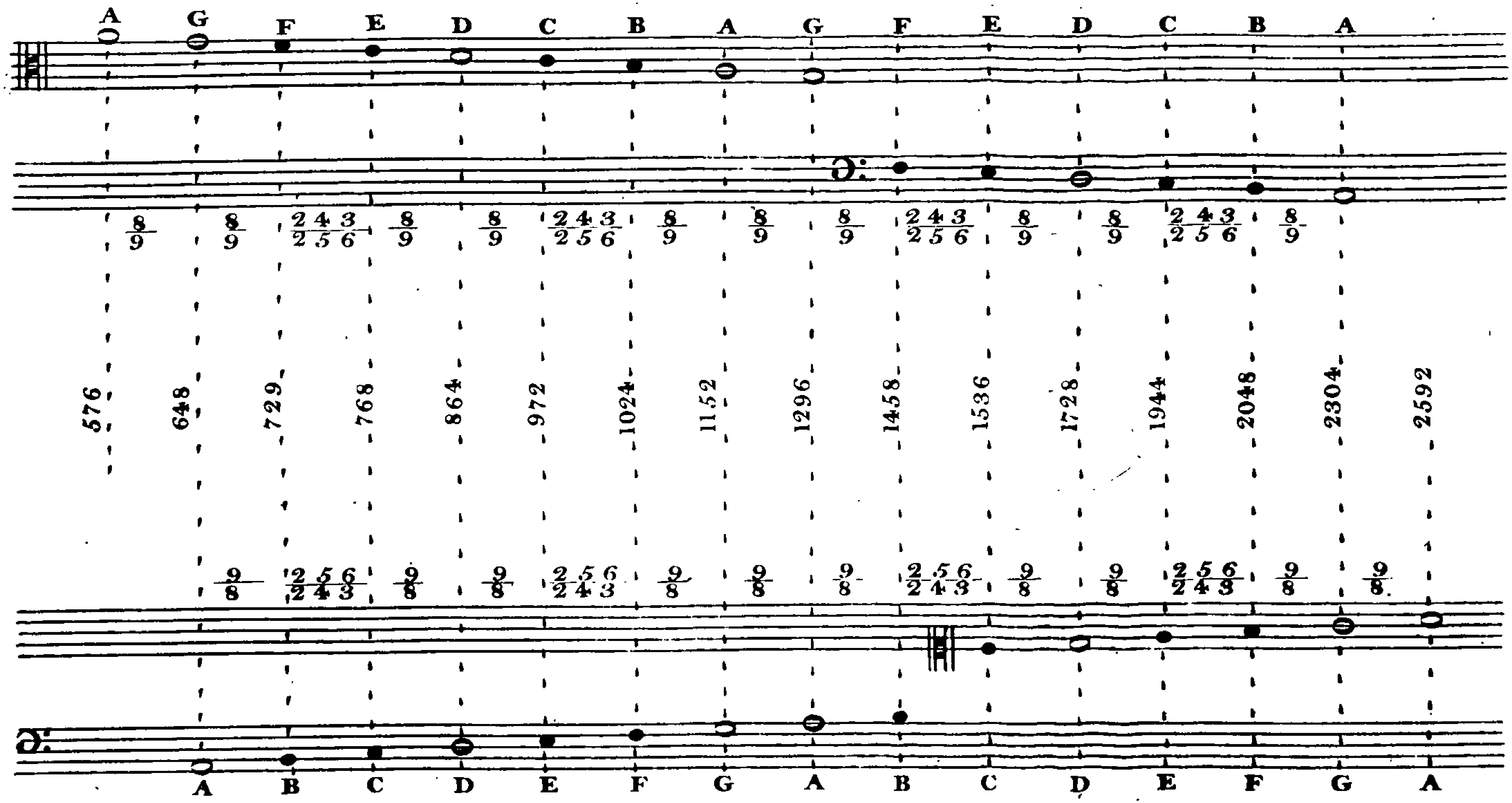
Plate XVIII.

DIAGRAMS OF GAUDENTIUS.

FIRST DIAGRAM.					SECOND DIAGRAM.	
		576	*Nete hyperbolæon*	2592		
9	8				9	8
		648	*Hyperbolæon diatonos*	2304		
9	8				9	8
		729	*Trite hyperbolæon*	2048		
256	243				256	243
		768	*Nete diezeugmenon*	1944		
9	8				9	8
		864	*Diezeugmenon diatonos*	1728		
9	8				9	8
		972	*Trite diezeugmenon*	1536		
256	243				256	243
		1024	*Paramesos*	1458		
9	8				9	8
		1152	*Mese*	1296		
9	8				9	8
		1296	*Meson diatonos*	1152		
9	8				9	8
		1458	*Parypate meson*	1024		
256	243				256	243
		1536	*Hypate meson*	972		
9	8				9	8
		1728	*Hypaton diatonos*	864		
9	8				9	8
		1944	*Parypate hypaton*	768		
256	243				256	243
		2048	*Hypate hypaton*	729		
9	8				9	8
		2304	*Proslambanomenos*	648		

Inversion of the Scales.

First Diagram.



Second Diagram.

Inversion of the Scales.

Plate XIX.

First Diagram.

Roots, & their Octaves.

Roots		Harmonics						
1	B	4					1024	2048
	G	5						
	E	6						
3	E	12				768	1536	
	C	15						
	A	18						
9	A	36				1152	2304	
	F	45						
	D	54						
27	D	108			864	1728		
	B♭	135						
	G	162						
81	G	324		648	1296	2592		
	E♭	405						
	C	486						
243	C	972		972	1944			
	A♭	1215						
	F	1458						
729	F	2916	729	1458				
	D♭	3645						
	B♭	4374						

SECOND DIAGRAM.
Roots and their Octaves.

Roots	Harmonics								
	F♯	4374							
	D♯	3645							
729	B	2916	729	1458					
	B	1458							
	G♯	1215							
243	E	972		972	1944				
	E	486							
	C♯	405							
81	A	324		648	1296	2592			
	A	162							
	F♯	135							
27	D	108			864	1728			
	D	54							
	B	45							
9	G	36				1152	2304		
	G	18							
	E	15							
3	C	12				768	1536		
	C	6							
	A	5							
1	F	4					1024	2048	

Tetrachord first Diagram.

fig:1

		$\frac{16}{15}$	$\frac{10}{9}$	$\frac{9}{8}$
	B	C	D	E
	32	30	27	24
	G	A	A	A
	40	36	36	36
	E	E	F	E
	48	48	45	48
F.B.	E	A	D	A
	96	72	54	72
Roots	1	3	9	3

Tetrachord second Diagram

fig:2

		$\frac{15}{16}$	$\frac{9}{10}$	$\frac{8}{9}$
	F	E	D	C
	64	60	54	48
	C	C	B	C
	48	48	45	48
	A	G	G	G
	40	36	36	36
F.B.	F	C	G	C
	32	24	18	24
Roots	1	3	9	3

The Scale, or Diatonic Octave in the first Diagram.

fig:3 (Tetrachord)

	A	$\frac{9}{10}$ G	$\frac{8}{9}$ F	$\frac{15}{16}$ E	$\frac{8}{9}$ D	$\frac{9}{10}$ C	$\frac{15}{16}$ B	$\frac{8}{9}$ A
	36	40	45	48	54	60	64	72
	$\frac{2}{1}$	$\frac{6}{5}$	$\frac{6}{5}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{6}{5}$	$\frac{3}{2}$	$\frac{2}{1}$
F.B.	72.A	48.E	54.D	72.A	108.D	72.A	96.E	144.A
Roots	3	1	9	3	9	3	1	3

The Scale, or Diatonic Octave, in the Second Diagram.

fig:4 (Tetrachord)

	C.	$\frac{9}{8}$ D.	$\frac{10}{9}$ E.	$\frac{16}{15}$ F.	$\frac{9}{8}$ G.	$\frac{10}{9}$ A.	$\frac{9}{8}$ B.	$\frac{16}{15}$ C.
	24	27	30	32	36	40	45	48
	$\frac{2}{1}$	$\frac{3}{2}$	$\frac{5}{4}$	$\frac{2}{1}$	$\frac{3}{2}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{2}{1}$
F.B.	12.C	18.G	24.C	16.F	24.C	32.F	36.G	24.C
Roots	3	9	3	1	3	1	9	3

First Diagram.

Tetrachords Conjoined.

Fig. 1.

Antecedent. Mean. Consequent.

	16/15	10/9	9/8	16/15	10/9	9/8	16/15	10/9	9/8	
	B	C	D	E	F	G	A	B♭	C	D
	512	480	432	384	360	324	288	270	243	216
	2/3	5/6	1/2	2/3	5/6	1/2	2/3	5/6	1/2	2/3
	768	576	864	576	432	648	432	324	486	324
Fundamental Bases	E	A	D	A	D	G	D	G	C	G
Roots.	1	3	9	3	9	27	9	27	81	27

First Diagram.

Tetrachords Disjoined.

Fig. 3.

	16/15	10/9	9/8	1/1	9/8	16/15	10/9	9/8	1/1	9/8	16/15	10/9	9/8	
	A	B♭	C	D	D	E	F	G	A	A	B	C	D	E
	288	270	243	216	216	192	180	162	144	144	128	120	108	96
	2/3	5/6	1/2	2/3	1/2	2/3	5/6	1/2	2/3	1/2	2/3	5/6	1/2	2/3
	432	324	486	324	432	288	216	324	216	288	192	144	216	144
F.B.	D	G	C	G	D	A	D	G	D	A	E	A	D	A
R.	9	27	81	27	9	3	9	27	9	3	1	3	9	3

SECOND DIAGRAM.

TETRACHORDS CONJOINED.

FIG. 2.

ANTECEDENT: 15/16, 9/10, 8/9 — MEAN: 15/16, 9/10, 8/9 — CONSEQUENT: 15/16, 9/10, 8/9

	F	E	D	C	B	A	G	F#	E	D
	512	480	432	384	360	324	288	270	243	216
	$\frac{2}{1}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{2}{1}$
	256	384	288	192	288	216	144	216	162	108
FUNDAMENTAL BASSES	F	C	G	C	G	D	G	D	A	D
ROOTS	1	3	9	3	9	27	9	27	81	27

SECOND DIAGRAM.

TETRACHORDS DISJOINED.

FIG. 4.

15/16, 9/10, 8/9, 1/1, 8/9, 15/16, 9/10, 8/9, 1/1, 8/9, 15/16, 9/10, 8/9

	G	F#	E	D	D	C	B	A	G	G	F	E	D	C
	288	270	243	216	216	192	180	162	144	144	128	120	108	96
	$\frac{2}{1}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{2}{1}$
	144	216	162	108	144	96	144	108	72	96	64	96	72	48
F.B.	G	D	A	D	G	C	G	D	G	C	F	C	G	C
R.	9	27	81	27	9	3	9	27	9	3	1	3	9	3

Plate XXV.

First Diagram. Quintuple.

Communication between the minor and major Scales.

1st Sectn. Root 1.

Root 1.	B	16
	G	20
	E	24
Root 3.	E	48
	C	60
	A	72
Root 9.	A	144
	F	180
	D	216
Root 27.	D	432
	Bb	540
	G	648
Root 81.	G	1296
	Eb	1620
	C	1944
Root 243.	C	3888
	Ab	4860
	F	5832
Root 729.	F	11664
	Db	14580
	Bb	17496

2d Sectn. Root 5.

G. 20
E 24 Eb 25
C 30
C 60
A 72. Ab 75
F 90
F 180
D 216 Db 225
Bb 270
Bb 540
G 648 Gb 675
Eb 810
Eb 1620
C 1944 Cb. 2025
Ab 2430
Ab 4860
F 5832 Fb 6075
Db. 7290
Db 14580
Bb 17496 Bbb 18225
Gb 21870

3d Sectn. Root 25.

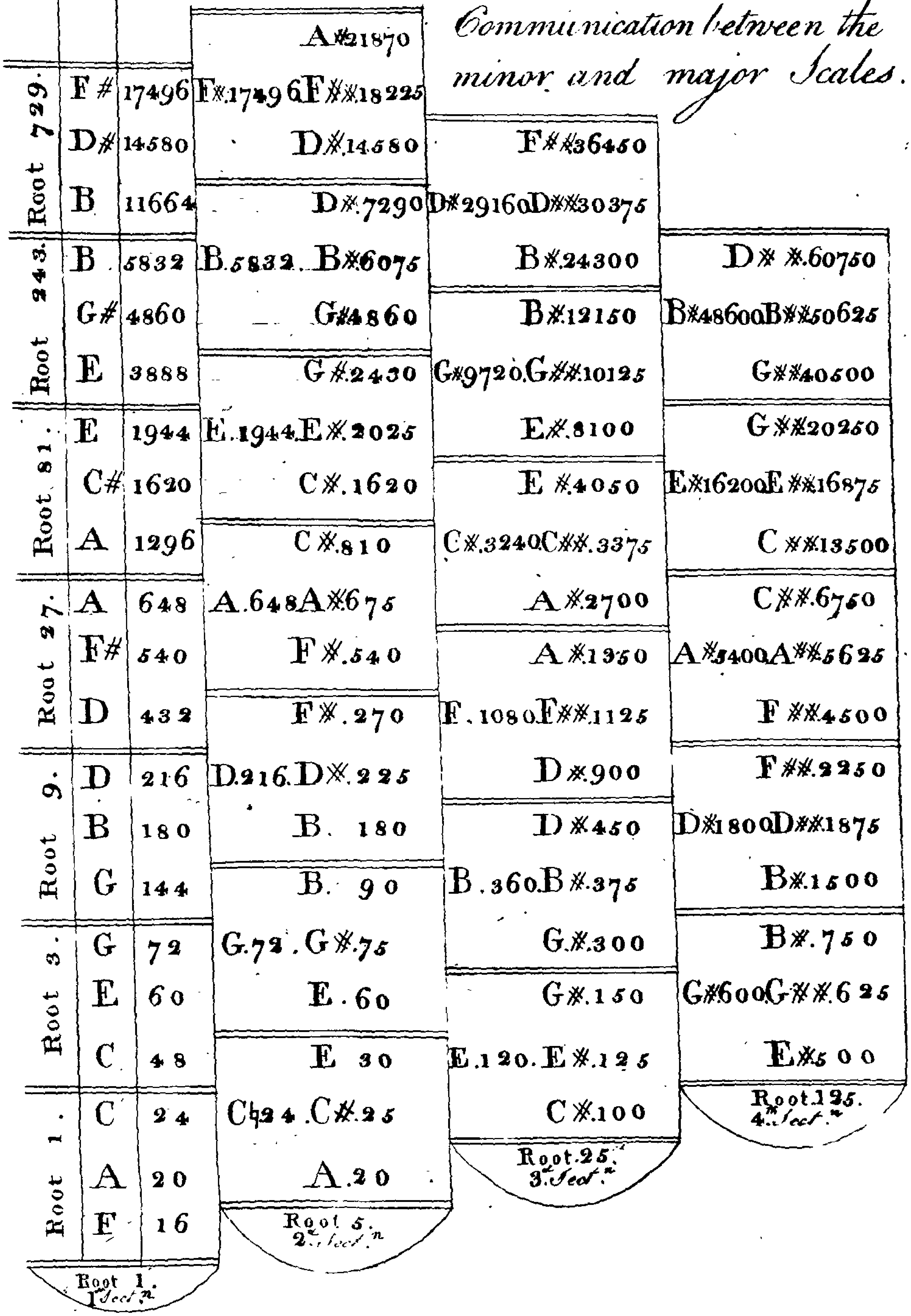
Eb. 100
C. 120. Cb. 125
Ab. 150
Ab. 300
F. 360. Fb. 375
Db. 450
Db. 900
Bb. 1050. Bbb. 1125
Gb. 1350
Gb. 2700
Eb 3240. Ebb. 3375
Cb. 4050
Cb. 8100
Ab. 9720. Abb. 10125
Fb. 12150
Fb. 24300
Db 29160. Dbb 30375
Bbb. 36450

4th Sectn. Root 125.

Cb. 500
Ab 600. Abb. 625
Fb. 750
Fb 1500
Db 1800. Dbb 1875
Bbb 2250
Bbb. 4500
Gb. 5400. Gbb 5625
Eb. 6750
Ebb. 13500
Cb. 16200. Cbb 16875
Abb 20250
Abb. 40500
Fb 48600. Fbb 50625
Dbb. 60750

Second Diagram. Quintuple.

Communication between the minor and major Scales.



The Three Terms, Or fundamental Basses, proper to form three Scales with Minor 3ds. & three with Major 3ds.

				Terms				
		A	3	3	30.	C		
1 Flat.	D	D	9	2	90.	F.	F.	1 Flat.
		G	27	1	270.	Bb		
		D	9	3	90.	F		
2 Flats.	G	G	27	2	270.	Bb	Bb.	2 Flats.
		C	81	1	810.	Eb		
		G	27	3	270.	Bb		
3 Flats.	C	C	81	2	810.	Eb	Eb.	3 Flats.
		F	243	1	2430.	Ab		

Fig. 1. Fig. 2.

The Three Terms, Or fundamental Basses, proper to form three Scales with Major 3ds. & three with Minor 3ds.

				Terms				
		E.	243	3	1620	C♯		
3 Sharps.	A	A.	81	2	540	F♯	F♯.	3 Sharps.
		D.	27.	1	180	B		
		A.	81.	3	540	F♯		
2 Sharps.	D	D.	27.	2	180	B	B.	2 Sharps.
		G.	9	1	60	E		
		D.	27.	3	180	B		
1 Sharp.	G	G.	9.	2	60	E	E.	1 Sharp.
		C.	3.	1	20	A		

Fig. 3. Fig. 4.

Plate XXVIII.

First Diagram.

Illustration of the Pythagorean Numbers.

Roots	Pythagorean Numbers		Suspension of the 4th	Suspension of the 9th	A 4th Term compounded.	A 4th Term double compounded
						F 9th
[illegible]	6	E		E 9th		
		C			D 7th	D 7th
Bass		A	A 8th	A 5th		
1	8	B			B 5th	B 5th
		G			G 3d	G 3d
Bass		E	E 5th		E 1	E 1
9	9	A				
		F		F 3d		
Bass		D	D 4th	D 1		
3	12	E				
		C				
Bass		A	A 1			

SECOND DIAGRAM.

Illustration of the Pythagorean Numbers.

	Roots		Suspension of ye 4th	Suspension of ye 9th	A third Term compounded	A third Term double compounded
						9th A ♮ or b
		G		G 9th		
		E			F b7th	F b7th
3	12	C	C 8th	C 5th		
		D			D 5th	D 5th
		B			B 3d #	B 3d #
9	9	G	G 5th		G 1	G 1
		C				
		A		A 3rd		
1	8	F	F 4th	F 1		
		G				
		E				
3	6	C	C 1			
	Pythagorean Numbers.					